Furrow Geometric Parameters

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Abstract: Furrow shape information is required for modeling and evaluating furrow irrigation. Currently used shape models assume the furrow perimeter is rigid so that only the flow depth increases with capacity. Actual furrow perimeters are not rigid and may widen as their capacity increases. If the furrow width increases proportionally with flow depth, the flow cross-sectional shape remains constant and only the size increases with capacity. This constant-shape model results in simple generalized relationships between the hydraulic and geometric parameters, which simplifies analysis of the complicated interactions that occur during furrow irrigation. The two shape models are compared conceptually and against field measurements. The rigid-perimeter model better matches field-measured furrow shapes and is easier to rationalize conceptually. However, both models match the important relationships between furrow geometric parameters and hydraulic parameters equally well. The most important relationship between flow area and uniform flow section factor is insensitive to both the model and shape. The predictions of both models are more sensitive to the furrow top width-to-flow depth ratio than to shape.

Introduction

Furrow geometry parameters are required for modeling and evaluating furrow irrigation. Cross-sectional area is required to calculate surface storage and flow velocity. Flow depth is required to calculate water surface elevation and thus the friction slope used in zero-inertia and hydrodynamic surface hydraulics models. Infiltration rate has been related to wetted perimeter. Hydraulic radius is needed to calculate tractive force for erosion models.

Furrow cross-sectional shape models are commonly used to interrelate the various geometric parameters. Surface flow equations can then relate the geometric parameters to the hydraulic parameters. The most commonly used equation is Manning's uniform flow relationship:

$$Q = \left(\frac{1}{n}\right) A R^{2/3} S^{1/2} \quad (1)$$

where Q = flow rate (m/s); A = flow cross-sectional area (m^2); R = hydraulic radius = A/P (m); P = wetted perimeter (m); S = water-surface slope (m/m); and n = channel roughness coefficient (s/m$^{1/3}$). The geometric parameters in Manning's equation, $AR^{2/3}$, can be separated from the hydraulic parameters giving:

$$AR^{2/3} = \frac{Qn}{S^{1/2}} \quad (2)$$

Chow (1959) refers to the left side of this equation as the section factor for uniform flow. In this paper, the symbol F designates the section factor. If the geometric parameters can be related to the Manning section factor, F,

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Note. Discussion open until March 1, 1992. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on August 28, 1990. This paper is part of the ***Journal of Irrigation and Drainage Engineering***, Vol. 117, No. 5, September/October, 1991. ©ASCE, ISSN 0733-9437/91/0005-0613/$1.00 + $.15 per page. Paper No. 26221.

they can be quantified for any required hydraulic capacity determined by the right side of (2).

Rigid-Perimeter Models

Presently used shape models assume that the channel perimeter is rigid and the shape of the furrow does not change. As the channel capacity increases, the flow depth increases but the width at any height above the red remains constant. Fig. 1 depicts the flow cross-sectional shape of a rigid-perimeter shape with variation in section factor. The cross sections shown in the figure are similar to those that might be encountered at the inflow end, the midpoint, and the outflow end of a furrow. As the figure shows, with a rigid-perimeter furrow, although the shape of the furrow is constant, the shape of the cross-sectional flow area changes.

Several rigid-perimeter furrow cross-sectional shapes have been used. Simple geometric shapes such as rectangles, triangles, or trapezoids are sometimes assumed. The Soil Conservation Service used trapezoidal shapes to generate relationships between geometric parameters for their furrow irrigation design procedure ("Furrow" 1983).

The most commonly used shape model since the availability of computers is the power function (Fangmeier and Ramsey (1978); Elliott et al. (1983); Trout (1983)):

$$w = b \cdot d^h \quad \text{(3)}$$

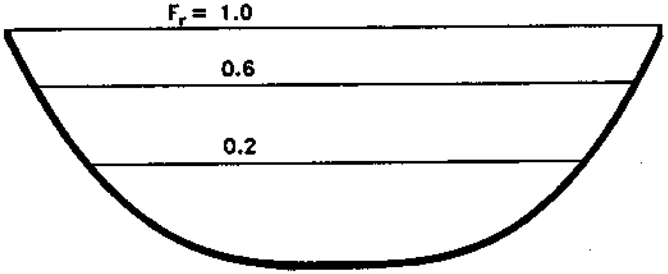

FIG. 1. RP Power Function Furrow Cross Sections at Three Section Factors (F_r = Relative Section Factor, h = 0.3, W/D = 3 at F_r = 1)

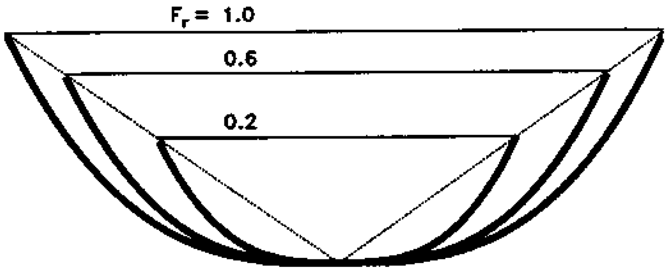

FIG. 2. CS Furrow Cross Sections at Three Section Factors (F_r = Relative Section Factor, h = 0.3, k = 3)

where d = height above the furrow bed; w = furrow width, and b and h = empirical coefficients.

The furrow shape in Fig. 1 is defined by a power function. This equation can model most symmetrical furrow shapes quite well. The coefficients can be determined either by least-squares regression of logarithmically transformed channel perimeter measurements or from values for channel width at two depths. The area, a, as a function of height above the bed, is determined by integration of (3) and is also a power function

$$a = \left[\frac{b}{(h+1)}\right] d^{(h+1)} \quad \text{(4)}$$

However, the perimeter, p, can only be determined by numerical procedures such as numerical integration of the line integral of $w/2$ as a function of d [(3)]. This is the main drawback to the shape. Some researchers have concluded that, in the range over which flow depths usually vary, the wetted perimeter can also be adequately represented by a power function of the same form as (3) (Fangmeier and Ramsey, 1978; Elliott et al. 1983).

$$p = g \cdot d^u \quad \text{(5)}$$

where g and u = empirical coefficients usually determined by curve fitting the numerically generated data. Although generating this approximation is numerically complex, it allows relationships to be derived among flow depth, D, cross-sectional flow area, A, and the wetted perimeter, P; and the Manning section factor, F

$$D = \left[g^{2/3}\left(\frac{h+1}{b}\right)^{5/3} F\right]^{3/5(h+1)-2u} \quad \text{(6)}$$

$$A = \left(\frac{h+1}{b}\right)^{2/5(h+1)/u-2} (g^{2/3}F)^{3/5-2u/(h+1)} \quad \text{(7)}$$

$$P = \left[\frac{(h+1)g^{(h+1)/u}}{b} F^{3/5}\right]^{5/5(h+1)/u-2} \quad \text{(8)}$$

Non-rigid Furrow Perimeters

Irrigation furrows are constructed in soil and thus their perimeters are not rigid but can be changed by the flowing water. The process of wetting dry soil and the shear of the flowing water detaches particles that move both with the flow and by gravity (sloughing). Fine particles are often washed away while coarser particles accumulate in the channel bed. Thus, furrow shapes change from the original mechanically formed shape toward a more hydraulically stable shape.

The eventual stable furrow shape is dependent on the cohesiveness of the soil, the size and stability of soil aggregates, and the magnitude and distribution of hydraulic shear forces (Chow 1959; Foster and Lane 1983; Lane and Foster 1980). In many soils, erosion from the side walls causes the channel to widen as the flow depth increases to accommodate more flow (or decreased slope), thus maintaining roughly the same flow cross-sectional shape and only changing size. Likewise, if the slope increases, flow depth and area decreases and flow velocity increases, which erodes the bed and results in a smaller cross section with a narrower perimeter. Fig. 2 shows a constant-shape furrow in which the perimeter becomes wider as the flow gets deeper.

The objective of this paper is to compare the constant-shape furrow geometry model with the rigid-perimeter model. The models are evaluated both conceptually and with field data.

CONSTANT-SHAPE MODEL

Fig. 2 shows furrow cross sections with constant shape. The shape shown is defined by a power function, like Fig. 1, but as the section factor changes, the width and depth change proportionately, resulting in a constant-flow cross-sectional shape (as compared to a constant perimeter shape). This is equivalent to a simple scale change such as would be produced by drawing the shape on the rubber sheet and stretching the sheet equally in all directions. Because the top width-to-flow depth ratio is constant, the changing top widths inscribe a triangle. Because width changes with flow depth, the depth change with an equivalent section factor change is less than with the rigid shape shown in Fig. 1. Also, since the shape is constant, the hydraulic efficiency of the cross section does not vary with capacity.

For a channel that maintains a constant shape, the width at any elevation above the bed, w, relative to the top width, W, is a function of the elevation above the bed, d, relative to the flow depth, D; and the top width-to-flow depth ratio, W/D, is constant. Thus, as the size increases, the width at any relative elevation increases proportionally with the flow depth. For example, a power function cross section with a constant shape is described by:

$$\frac{w}{W} = \left(\frac{d}{D}\right)^h \quad (9)$$

or:

$$w = k \cdot D \left(\frac{d}{D}\right)^h \quad (10)$$

where $k = W/D$ and h = an empirical constant as defined in (3). Constant-shape profile equations for several commonly used channel shapes are given in Table 1. Note that the triangle [(3) or (10) with $h = 1$] is the only rigid cross section that also maintains a constant shape.

The channel area can be determined by integrating the width equation. For the power function shape, the cross-sectional area is given by

$$A = \left(\frac{k}{(h + 1)}\right) D^2 \quad (11)$$

Because width is proportional to flow depth, area is proportional to D^2 for constant-shape cross sections and can be represented by $A = k_a D^2$ with k_a the proportionality constant.

Because the shape remains constant and only the size changes, the wetted perimeter is also proportional to flow depth and can be represented by the equation $P = k_p D$. For the power-function shape, the perimeter can be calculated from the line integral of the d versus w relationship:

TABLE 1. Constant-Shape Furrow Geometric Equations ($k = W/D$; $k_a = A/D^2$; $k_p = P/D$)

Shape (1)	Relative width, w/W (2)	k_a (3)	k_p
Triangle[a]	(d/D)	$k/2$	$2\sqrt{(k/2)^2 + 1}$
Trapezoid[b]	$1 + 2z[1 - (d/D)]/W$	$k - z$	$k - 2z + 2\sqrt{(z^2 + 1)}$
Semi-elipse	$[2(d/D) - (d/D)^2]^{1/2}$	$\pi k/4$	$\pi(k^2/8 + 1/2)^{1/2}$
Parabola[c]	$(d/D)^{1/2}$	$(2/3)k$	$(4 + k^2/4)^{1/2} + k^2/8 \ln\left[\frac{2 + (4 + k^2/4)^{1/2}}{k/2}\right]$
Power curve[d]	$(d/D)^h$	$k/(h + 1)$	$2\int_0^1 [1 + (2/kh)^2 r^{(2/h-2)}] dr$

[a] $k = 2z$, where z = 1/side slope.
[b] z = 1/side slope.
[c] $k = 4(pD)^{1/2}$, where p = focus of the parabola.
[d] k_p = line integral of $d(w/2)$.

$$P = 2D\int_0^1 \left[1 + \left(\frac{2}{kh}\right)^2 r^{(2/h-2)}\right]^{1/2} dr \quad (12)$$

The integral in (12) is not a function of depth but varies only with the shape parameters k and h. Thus it need be calculated only once for a given shape and an approximation of the p versus d relationship [such as (5)] is not required. The proportionality constants k_a and k_p for several cross-sectional shapes are given in Table 1. Note that k, k_a, and k_p are dimensionless coefficients [unlike b in (3) and g in (5)] and thus are independent of the units used.

Since $A = k_a D^2$ and $P = k_p D$, the Manning section factor for uniform flow is given by:

$$F = AR^{2/3} = \frac{A^{5/3}}{P^{2/3}} = \left(\frac{k_a^{5/3}}{k_p^{2/3}}\right) D^{8/3} \quad (13)$$

The constant-shape assumption results in the section factor being proportional to $D^{8/3}$, regardless of the shape. Similarly, if the Chezy uniform-flow formula is used, the section factor is proportional to $D^{5/2}$.

These constant-shape relationships greatly simplify the determination of channel geometric parameters. Since both area and wetted perimeter and thus section factor are all proportional to depth of a power, D, A, and P can all be determined directly knowing the furrow shape, represented by constants k_a and k_p, and the section factor.

$$D = \left(\frac{k_p^{1/4}}{k_a^{5/8}}\right) F^{3/8} = K_D F^{3/8} \quad (14)$$

$$A = \left(\frac{k_p^{1/2}}{k_a^{1/4}}\right) F^{3/4} = K_A\, F^{3/4} \quad (15)$$

$$P = \left(\frac{k_p^{5/4}}{k_a^{5/8}}\right) F^{3/8} = K_p\, F^{3/8} \quad (16)$$

These relationships are similar to but less complex than (6)–(8) for the rigid-perimeter shape with the power-function–wetted-perimeter approximation.

Since the section factor is equal to $Qn/S^{1/2}$, if k_p and k_a are known, the geometric parameters can be directly related to flow rate, slope, and roughness. Even if the actual shape is not known, because the exponent values are fixed, the relative relationships will remain constant. In terms of relative changes:

$$\frac{\Delta D}{D} = \left(\frac{3}{8}\right)\left(\frac{\Delta F}{F}\right) = \left(\frac{3}{8}\right)\left(\frac{\Delta Q}{Q}\right) = \left(\frac{3}{8}\right)\left(\frac{\Delta n}{n}\right) = \left(\frac{-3}{16}\right)\left(\frac{\Delta S}{S}\right) \quad (17)$$

$$\frac{\Delta A}{A} = \left(\frac{3}{4}\right)\left(\frac{\Delta F}{F}\right) = \left(\frac{3}{4}\right)\left(\frac{\Delta Q}{Q}\right) = \left(\frac{3}{4}\right)\left(\frac{\Delta n}{n}\right) = \left(\frac{-3}{8}\right)\left(\frac{\Delta S}{S}\right) \quad (18)$$

$$\frac{\Delta P}{P} = \left(\frac{3}{8}\right)\left(\frac{\Delta F}{F}\right) = \left(\frac{3}{8}\right)\left(\frac{\Delta Q}{Q}\right) = \left(\frac{3}{8}\right)\left(\frac{\Delta n}{n}\right) = \left(\frac{-3}{16}\right)\left(\frac{\Delta S}{S}\right) \quad (19)$$

where $\Delta x/x$ denotes a relative change in the parameter value. Likewise, relative changes in the average flow velocity, V, and average shear-per-unit wetted perimeter, T, parameters important to erosion and sediment transport (Kemper et al. 1985), can be calculated:

$$\frac{\Delta V}{V} = \left(\frac{1}{4}\right)\left(\frac{\Delta Q}{Q}\right) = \left(\frac{-3}{4}\right)\left(\frac{\Delta n}{n}\right) = \left(\frac{3}{8}\right)\left(\frac{\Delta S}{S}\right) \quad (20)$$

$$\frac{\Delta T}{T} = \left(\frac{3}{8}\right)\left(\frac{\Delta Q}{Q}\right) = \left(\frac{3}{8}\right)\left(\frac{\Delta n}{n}\right) = \left(\frac{3}{16}\right)\left(\frac{\Delta S}{S}\right) \quad (21)$$

With these relative relationships, if the value of the geometric parameter is known for any set of hydraulic parameters, it is easily determined for any other hydraulic parameter values. For example, if the flow area is known at the furrow inflow end, it can be calculated by (18) at any downstream location given the relative change in the hydraulic parameters at the two locations.

Field Evaluation Of Cross-Section Models

Many researchers have measured furrow cross-sectional profiles. However, most of the data reported in the literature are of furrow perimeter shape for a particular location and set of hydraulic conditions [e.g., Fangmeier and Ramsey 1978; Mostafazadehfard 1985), rather than cross-sectional flow shape, which requires flow depth or section factor information. This methodology assumes the perimeter is rigid. To compare the rigid-perimeter and constant-shape models, furrow geometric parameters (W, D, P, and A) are required for flow cross sections created over a wide range of

flow conditions. Elliott et al. (1980) present furrow flow depth and top width data pairs for a large number of irrigations and flow conditions. However, these data sets exhibited no relationships between the widely scattered values of these two parameters. Because of the lack of existing applicable data sets, furrow geometric data were collected to evaluate the models.

Procedure

Furrow cross-sectional flow shapes were measured in Colorado and Idaho under a wide range of flow rates and slopes (and thus section factors). Furrow perimeter and water-surface evaluations were measured with a profilometer ("Evaluation", 1989) with 10-mm-diameter pins on 20-mm centers. The measurements were made late in the irrigation events and thus represent final shapes under steady-flow conditions.

Colorado field data were collected during the initial two irrigations of the season on a Glenton loam soil at the Colorado State University Fruita Research Farm near Grand Junction. The field was cultivated between the irrigations. Furrow cross-sectional profile data were collected at 1/6, 1/2, and 5/6 the length from the inflow end of the 300-m long furrows. Three furrow inflow rates between 20 and 70 L/min were replicated six times during the first irrigation, producing 54 profile measurements. During the second irrigation, the same flow rates were replicated three times. Furrow slope was 0.01 m/m. The section factor ranged from 5,000 to 90,000 $mm^{8/3}$ (0.00005–0.0009 $m^{8/3}$).

Idaho field data were similarly collected on a Portneuf silt loam soil at the USDA-ARS Kimberly Research Station. Profiles were measured at 180-m-long furrows with a 0.007-m/m slope at three locations (1/6, 1/2, and 5/6 length) during the first and second irrigations on a field. The field was cultivated between the irrigations. Four flow rates between 20 and 45 L/min were applied to three replicates, resulting in 36 measurements during each irrigation. The section factor ranged from 1,000 to 22,000 $mm^{8/3}$.

Geometric data were collected during recirculating infiltrometer tests on three fields with a Portneuf soil at the Kimberly, Idaho, location over two years. The tests were designed to determine the effects of slope and flow rate on furrow infiltration and hydraulic parameters. Flow rates from 6 to 40 L/min were applied to furrows laid out in a radial pattern to create furrow slopes of 0.002 to 0.02 m/m. Thus, both slope and flow rate were varied widely in a small plot area, yielding 71 furrow sections, with section factors ranging from 2,000 to 35,000 $mm^{8/3}$. Furrow cross-sectional profiles were measured 2 m from the ends of the 6-m-long sections and the geometric parameters for the two measurements were averaged.

The perimeter profile and water-surface elevation data were converted to geometric parameters of flow depth, top width, wetted perimeter, and area by linearly interpolating between the measured profile elevations up to the water surface. The section factor was calculated for each profile. Visual evaluation of the profiles indicated they could be best matched by the power function shape. Thus, these parameters were logarithmically transformed and linearly regressed to develop best-fit power-function (PF) relationships between the parameters. Best-fit constant-shape (CS) regression relationships were also generated by linearly regressing the dependent variable against the independent variable taken to the appropriate power [see (10), (11), (12), (14), (15), and (16)] and forcing the intercept to zero. The CS regression relationship has only one coefficient since the exponent is fixed, and is not tied to any assumed furrow shape.

The rigid-perimeter (RP) model was derived for each data set from (4)–(8) using the coefficient and exponent of the W versus D PF regression relationship for b and h. Note that these coefficients were determined from top width and flow depth data pairs from many cross sections rather than by regressing perimeter width versus elevation above the bed for individual profiles. If the perimeter is rigid, the result will be the same except that the procedure used will determine coefficients for a composite profile. The g and u coefficients were determined by numerically integrating the W versus D regression relationship and regressing the calculated perimeter versus depth. A CS model was derived from (14), (15), and (16) using the coefficient values from the best-fit CS regression relationships between A and D and between P and D for k_a and k_p, respectively. The fits of the models were compared by calculating the average absolute deviation of the regression or model-predicted values from the measured values:

$$\text{err} = \text{avg}\left[\frac{\text{abs (predicted} - \text{measured)}}{\text{measured}}\right] \cdot 100 \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Results

Table 2 lists the coefficients for the best-fit PF regression equation, the derived RP power function model, the CS regression relationship, and the derived CS model. Figs. 3–7 show the relationships among the geometric parameters for the three data sets and the regression, RP, and CS models. The data scatter is typical for furrow cross-sectional measurements.

The CS model is based on the assumption that top width increases proportionally with the flow depth. Fig. 3 shows that although W does tend to increase with D, the measured relationship is less than proportional (depicted by the CS regression line) for all three data sets. The relationships shown could result from rigid-perimeter shapes with sloping sides at the water surface such as a power function (the PF regression curve) or a trapezoid (a straight line with a positive intercept). The data could also result from an increasing width shape with the increase less than proportional to depth. Whether the perimeter is rigid or widening cannot be determined from the data, but the RP power function relationship fits the data better than the CS relationship (Table 2). Note that the coefficient, c, and exponent, e, for the W versus D PF regression relationship in Table 2 are equivalent to b and h, respectively [(3)], and that c for the CS regression relationship is a best estimate of k for the CS model.

The P versus D data trends and scatter are similar to those shown in Fig. 3 for W versus D. This is expected since top widths averaged four-to-six times the flow depths and thus are the dominant dimension of the perimeter. The RP power function model coefficients listed in Table 2 were calculated from the line integral of the W versus D PF regression equations. These derived relationships fit the data nearly as well as the BF curve, supporting this commonly used procedure of approximating the P versus D relationship. The CS regression coefficients provide best estimates for k_p for the data sets.

Fig. 4 shows the measured cross-sectional flow area versus flow depth data. These data are best fit by power functions with exponents less than two, again resulting from the width increase less than that predicted by the CS model. For all three data sets, the coefficient and exponent of the RP model [$b/(h + 1)$ and $h = 1$ respectively using b and h from the W versus D PF regression relationship] are larger than those of the BF regression

TABLE 2. Regression and Model Coefficients and Error for Measured Geometric Data (y versus x Where $y = c \cdot x^e$; All Units in Meters)

Parameters		Colorado			Idaho			Infiltrometer		
yx (1)	Eq. (2)	c (3)	e (4)	err^a (5)	c (6)	e (7)	err^a (8)	c (9)	e (10)	err^a (11)
W versus D	PF regression	0.49	0.26	14	0.21	0.19	16	0.65	0.48	12
	CS regression	6.23	1.00	23	4.77	1.00	26	4.24	1.00	19
P versus D	PF regression	0.71	0.34	12	0.39	0.30	14	1.06	0.57	11
	RP model	0.74	0.35	12	0.62	0.43	14	1.10	0.59	11
	CS regression	6.88	1.00	21	5.56	1.00	22	4.96	1.00	16
A versus D	PF regression	0.24	1.19	16	0.10	1.09	20	0.38	1.45	17
	RP model	0.39	1.26	27	0.18	1.19	29	0.44	1.48	17
	CS regression	3.64	2.00	25	2.89	2.00	31	2.53	2.00	25
A versus P	PF regression	0.07	1.89	22	0.12	2.08	21	0.13	2.10	12
A versus F	PF regression	1.15	0.69	4	1.07	0.70	4	1.34	0.72	3
	RP model	0.98	0.67	4	1.07	0.70	6	1.22	0.71	3
	CS regression	1.84	0.75	5	1.75	0.75	4	1.71	0.75	3
	CS model	1.90	0.75	5	1.81	0.75	5	1.77	0.75	3
D versus F	PF regression	1.68	0.49	10	2.69	0.52	14	1.02	0.41	10
	RP model	2.09	0.54	16	4.48	0.59	19	2.01	0.48	10
	CS regression	0.67	0.38	13	0.72	0.38	17	0.77	0.38	10
	CS model	0.72	0.38	16	0.79	0.38	22	0.84	0.38	15
P versus F	PF regression	1.41	0.23	10	1.20	0.24	10	2.12	0.31	6
	RP model	0.96	0.19	11	1.18	0.25	13	1.66	0.28	7
	CS regression	4.76	0.38	13	4.16	0.38	11	3.94	0.38	7
	CS model	4.97	0.39	14	4.40	0.38	12	4.14	0.38	8

[a] Average absolute deviation (%).

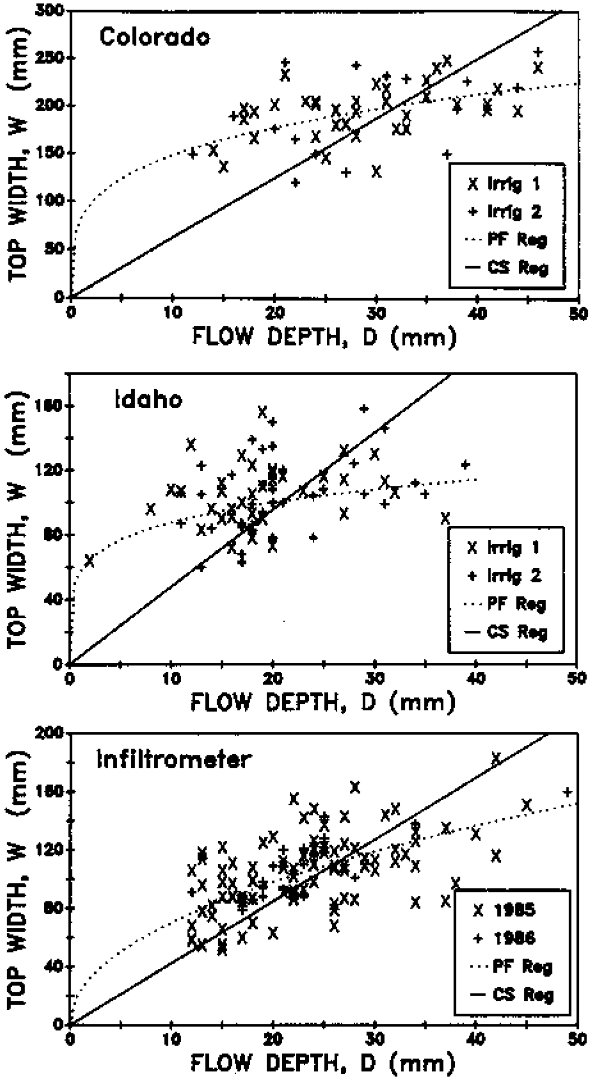

FIG. 3. Top Width versus Flow Depth for Three Data Sets Showing Measured Data and Regression Relationships

relationship, and result in predicted areas exceeding measured areas. The reason for this overprediction is not known. The coefficient of the CS regression relationship provides the best estimate for k_a.

The most important relationship for hydraulic modeling of furrow flow is flow area as a function of sector factor. As Fig. 5 and Table 2 show, the data scatter is much less than in the previous relationships and all models fit the data well. The CS regression fits the data nearly as well as the BF regression, indicating there is little disadvantage in fixing the exponent at

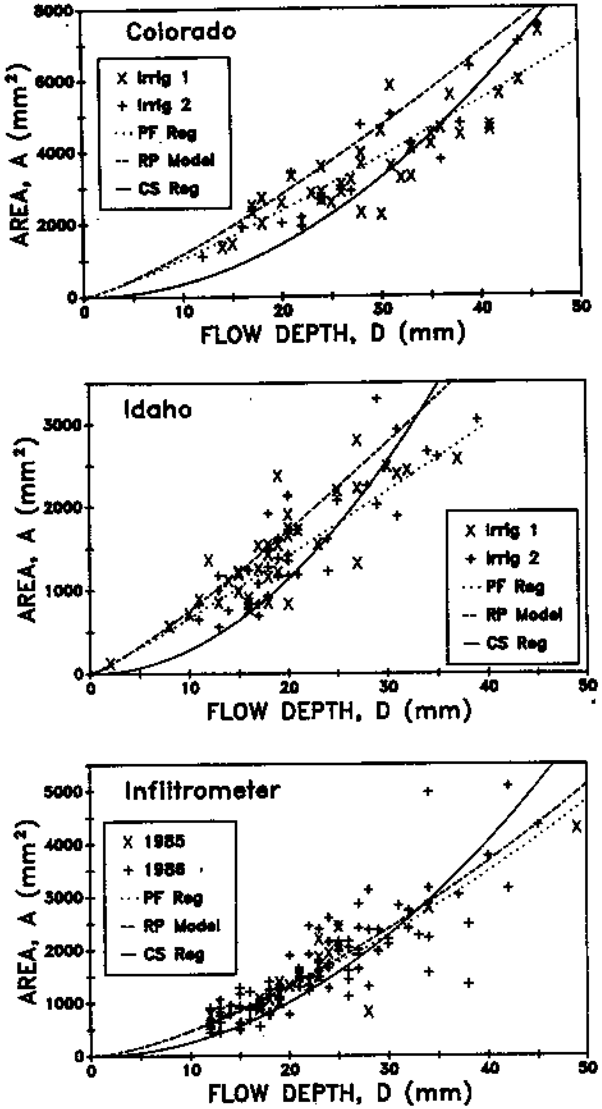

FIG. 4. Flow Area versus Flow Depth for Three Data Sets Showing Measured Data and Regression and Model-Predicted Relationships

0.75. The CS model [(15)], with k_p and k_a from the previous CS regression relationships, tends to slightly overestimate measured areas. The RP model [(7)], with b and h from the PF regression relationships, tends to underestimate areas by about the same amount.

Figs. 6 and 7 show that the data scatter is greater for the D versus F and P versus F relationships than the A versus F relationship, but smaller than for the "shape" relationships (W, P, and A versus D). This is expected from

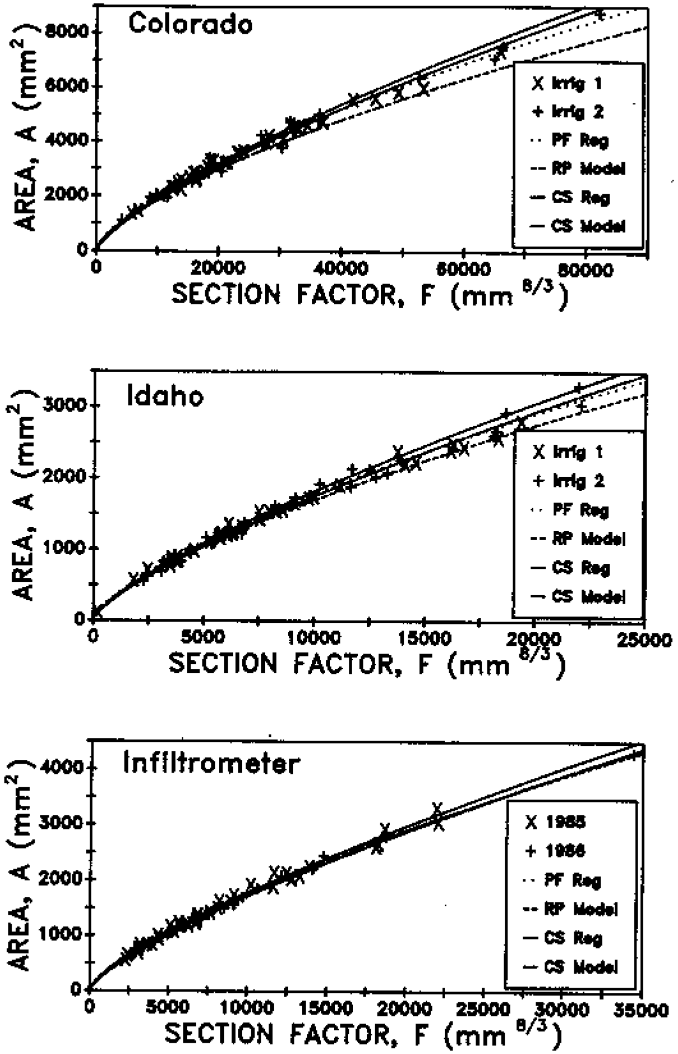

FIG. 5. Flow Area versus Section Factor for Three Data Sets Showing Measured Data and Regression and Model-Predicted Relationships

the parameter interrelationships. The D versus F best-fit regression exponents are consistently larger and the P versus F exponents consistently smaller than the 3/8 of the CS model for all three data sets. However, the CS regression fits the measured parameters nearly as well as the PF regression (two percentage points larger average error) so fixing the exponent at 3/8 is not a great disadvantage. The CS model tends to overestimate the data but the error is only slightly larger than with the RP model. Average deviations are typically in the range of 10–15% for both relationships and all three data sets. The reason the CS model coefficient exceeds the CS regression coefficient in all cases by 5–10% is not known.

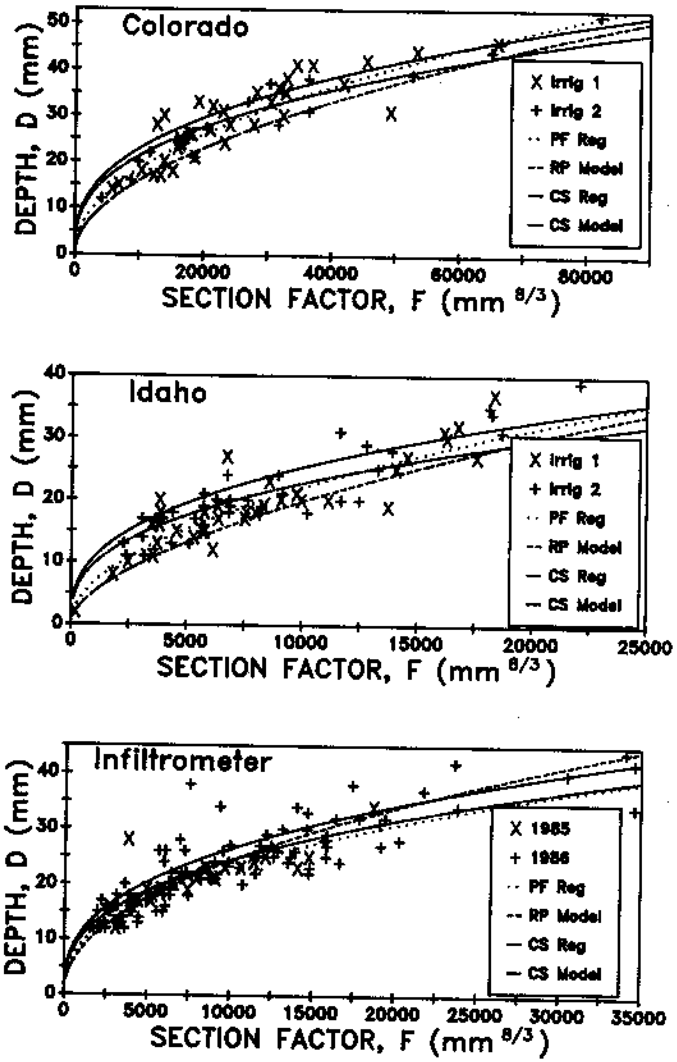

FIG. 6. Flow Depth versus Section Factor for Three Data Sets Showing Measured Data and Regression and Model-Predicted Relationships

ANALYSIS

Choice of a furrow shape model should be based on three criteria:

1. Conceptual authenticity.
2. Accuracy.
3. Simplicity

Conceptual Authenticity

The conceptual bases for the two discussed models are quite different. Cases can be made for both concepts and neither are valid for all conditions.

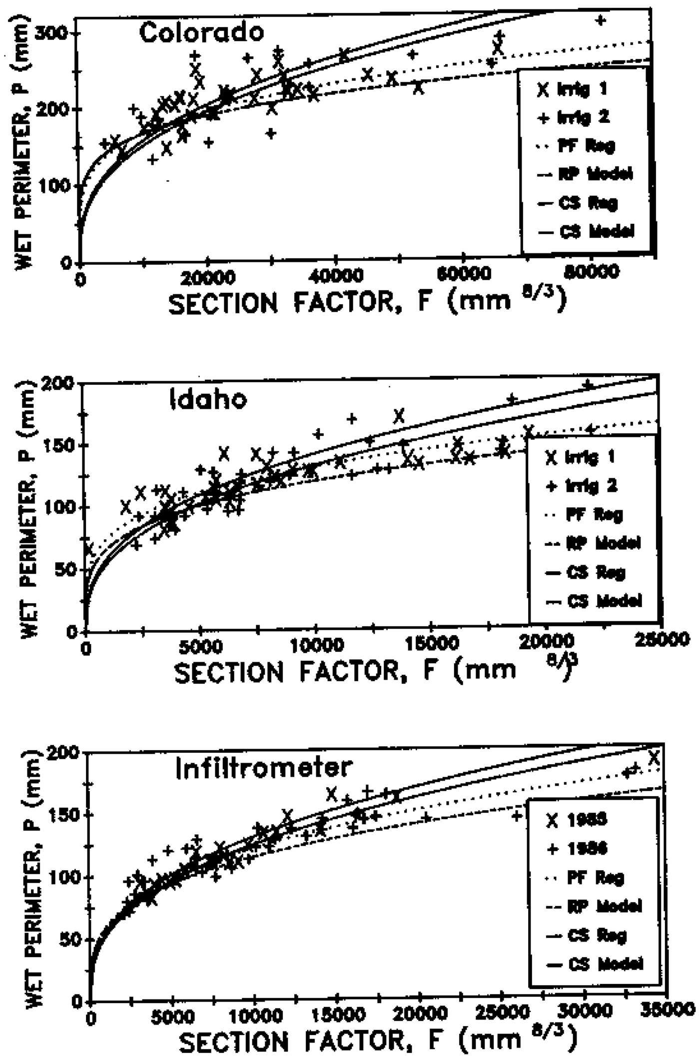

FIG. 7. Wetted Perimeter versus Section Factor for Three Data Sets Showing Measured Data and Regression and Model Predicted Relationships

Which is the better approximation will depend on the soil type, soil conditions, and hydraulic conditions, as discussed in the introduction. Furrows on flat slopes in stable soils will tend to maintain a fixed RP while steeper furrows with more erosive flows in more erodible soil will tend toward the CS model.

A conceptual problem with the CS model is that the perimeter width is predicted to shrink as the flow rate decreases during a cutback or recession phase. The rationale that larger flows can create wider furrows does not hold in reverse for flow rate decreases. Therefore, if accurate prediction of

flow depth or wetted perimeter is required during periods of decreasing section factor with time, the CS model should not be used. (Predicted flow area is insensitive to the model used.)

A possible conceptual difference between the CS and the RP model is the choice of the reference elevation from which the water surface elevation, and thus the water friction slope, is calculated. In an RP channel, the bed is assumed stable and thus is the logical reference point. With a non-rigid perimeter, the bed reference can also be used but the rationale is less clear. Conceptually, and in reality, the bed may erode downward as the flow depth increases. The bed could also accumulate sediment as the channel walls slough and erode and thus increase in elevation. However, since the stability and elevation change of the channel bed will vary with the flow conditions and be difficult to predict, maintaining the original bed elevation as the reference is a reasonable assumption for both models.

Accuracy

The accuracy of a model depends on two factors: (1) How well the mathematical model matches the physical model (the real world); and (2) how accurately the coefficients are determined and the sensitivity of the results to those coefficients. The collected data indicated that the RP model matched field furrow shape measurements better than the CS model, even with the fairly erosive soils at the measurement sites. However, furrow shape per se is not an important parameter in surface irrigation evaluation and prediction. Of importance are the relationships between channel flow area, flow depth, and wetted perimeter and the hydraulic parameters represented by the section factor. Table 2 and Figs. 5, 6,and 7 show that the CS cross section model fits the measured relationships between the geometric parameters and section factor nearly as well as the RP power function model.

Relative Relationships

Both models predict the flow depth, area, or wetted perimeter at a given section factor equally well if the same shapes are used (for example, a power function) and the shape coefficient values are correct. Where the models differ is in their ability to predict relative changes in the geometric parameters with changes in the hydraulic parameters (section factor). These relative relationships are determined by the exponents of the relationships [(6)–(8) and (14)–(16)]. The CS shape model exponents are fixed (independent of actual shape) while the RP relationship exponents vary with shape.

To compare these relative relationships, wetted perimeters, flow areas, and section factors were calculated over a wide range of depths for several RP power function shapes. These parameters were then normalized relative to their values at a depth of one unit, so that b is equal to the top width-to-depth ratio at $F = 1$. Constant shape relative relationships were calculated directly from (14)–(16) with the coefficients set to one.

Flow area is the most important parameter in surface hydraulics models since it determines the water volume stored on the surface. Fig. 5 indicated, and Fig. 8 verifies, that the A vs. F relationship is very insensitive to both the model and the actual shape. The theoretical limits of the exponent of the generated A versus F relationship for concave upward RP shapes ($h < 1$) is 0.6 (for $u = 0$) and 0.75 (for $u = 1$, $h = 1$). This exponent range results in less than a 10% variation in predicted area over a wide section factor range. The CS model, essentially a RP triangle, represents the upper limit of this exponent range.

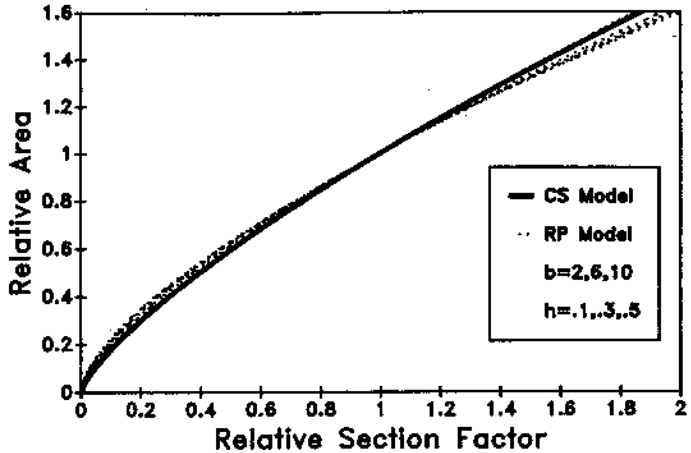

FIG. 8. Relative Area versus Relative Section Factor for CS Model and Range of RP Power Function Shapes

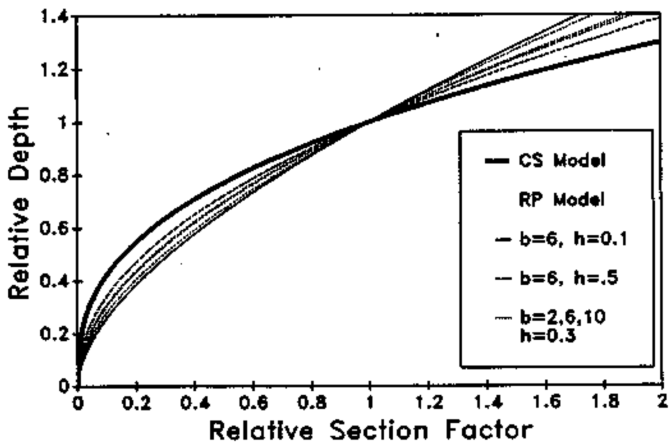

FIG. 9. Relative Flow Depth versus Relative Section Factor for CS Model and Range of RP Power Function Shapes

This insensitivity occurs because the largest component of the section factor is the flow area ($F = A^{5/3}/P^{2/3}$) and the other component, wetted perimeter, is positively related to area. In spite of the variation in exponents of the shape relationships, the A versus P regression relationships (Table 2) all have exponents near 2, which results in A versus F relationship exponents near 0.75. Thus, when flow area (surface storage volume) is the only important geometric parameter, model choice should be based on factors other than predictive capability.

Figs. 1 and 2 indicate that the change in normal flow depth with a change in section factor in a CS channel will be less than that in a RP channel. The

BF regression exponents were always larger than the 3/8 CS model exponent (Table 2). Fig. 9 supports these results. The CS model has a slower rate of depth change than all the tested RP shapes. Depths change most rapidly with section factor for RP shapes with small h values (rectangular shapes) and predicted depth changes are more sensitive to h values than the relative channel width-to-depth ratio, b.

Flow depth is used to predict water-surface slope in hydrodynamic and zero-inertia surface hydraulics models. When accurately predicting flow depth is important (very small furrow slopes), the results may be influenced by model choice and coefficient values. However, the magnitude of the resulting advance rate error will be reduced by the self-compensating nature of slope/flow rate interactions.

Fig. 10 shows that the CS model will generally predict a greater change in wetted perimeter with section factor than RP models. The exponents smaller than 3/8 of the PF regression and RP models (Table 2) support this conclusion. The CS model wetted perimeter varies more because width increases proportionally with depth and the top width-to-depth ratios are longer than one. Fig. 10 also shows that the relative relationship is sensitive to both the b and h coefficients for the RP model. The wetted-perimeter variation with section factor [exponent of (8)] increases with wider and more triangular channels (large b and h values).

Wetted perimeter is not presently used in most surface irrigation models. However, several researchers have proposed that infiltration varies directly with wetted perimeter (Fangmeier and Ramsey 1978; Samani 1983). The constant-shape model predicts a greater variation of wetted perimeter, and thus water application, with flow rate (and thus location along a furrow) than the RP model. For example, for a furrow with a 70% flow rate reduction from the inflow to the outflow end, the CS model predicts a wetted-perimeter reduction of 35% while a typical RP model ($b = 6$, $h = 0.3$) predicts a 20% reduction. Such differences can significantly impact predicted water-distribution uniformity. The infiltration–wetted-perimeter relationship is not yet well quantified but is usually projected as less than proportional, resulting in water-application errors smaller than wetted-perimeter errors.

Shape-Parameter Determination

The deviations between the measured geometric parameters and the best-fit regression-relationship predictions are larger than the differences between the predictions of the best-fit regression and the two shape models (Table 2). This indicates that error resulting from inaccurately estimated model coefficients will likely be larger than the error resulting from model inaccuracy. Accurate coefficient determination is thus more critical to accurate parameter prediction than model selection. Shape coefficient errors are due not to measurement imprecision but to furrow shape variability. Thus, a large number of shape measurements are required to establish confidence in a representative coefficient value (mean). Model accuracy is generally improved for a given time investment by quickly measuring shape coefficients at several locations rather than accurately determining the shape at a few locations.

Figs. 11–13 show the variation in the constant-shape model geometric coefficient values [K_D, K_A, and K_P in (14)–(16)] with the shape coefficients, k and h. All three geometric coefficients are much more sensitive to k values than to h values. Use of power function shapes with h values from 0.1 to 0.5, ellipses, or parabolas (a power function with $h = 0.5$) had only small

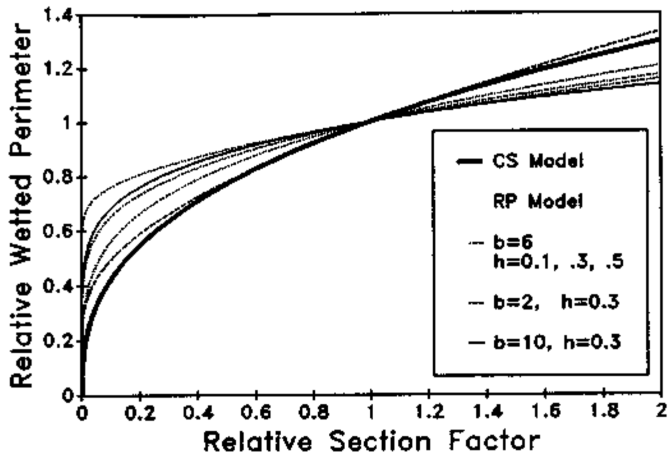

FIG. 10. Relative Wetted Perimeter versus Relative Section Factor for CS Model and Range of RP Power Function Shapes

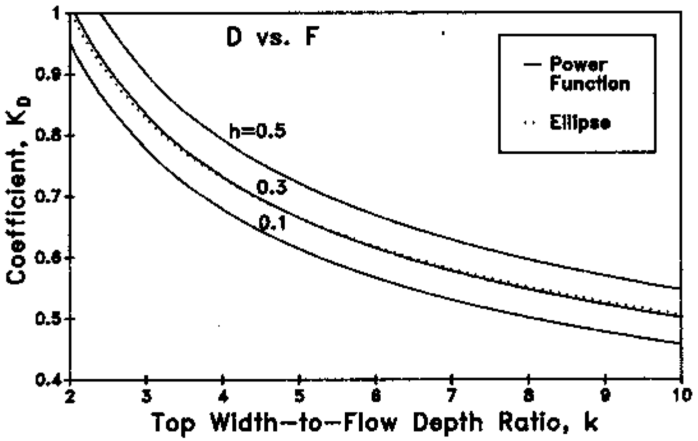

FIG. 11. CS *D* versus *F* Coefficient Variation with *k* and *h*

effects on the coefficients. Trapezoids would also produce good results and be relatively insensitive to side wall slopes. Consequently, the shape chosen in the CS model should be based on simplicity rather than accuracy, and there is generally no need to determine actual profile shapes. More important is to accurately determine the furrow top width-to-depth ratio, k. A sensitivity analysis for the RP model shape coefficients was not carried out because both the geometric parameter coefficient and exponent vary. However, the conclusions should be similar to those for the CS model.

With the RP power function model, four shape coefficients, b, h, g, and u, are required. The profile shape parameters, b and h, are usually determined by regressing log-transformed w and d data for individual profiles

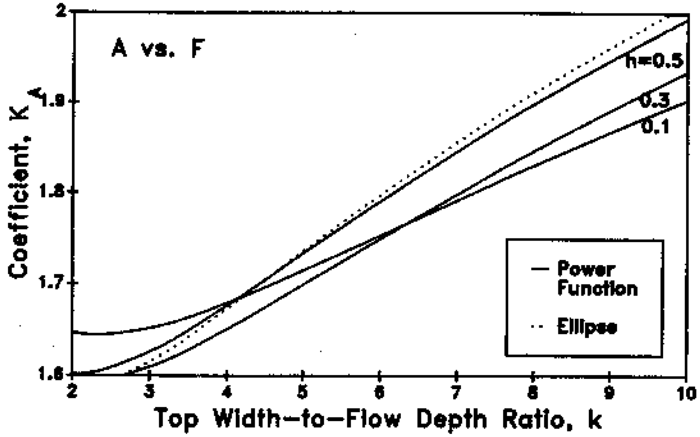

FIG. 12. CS *A* versus *F* Coefficient Variation with *k* and *h*

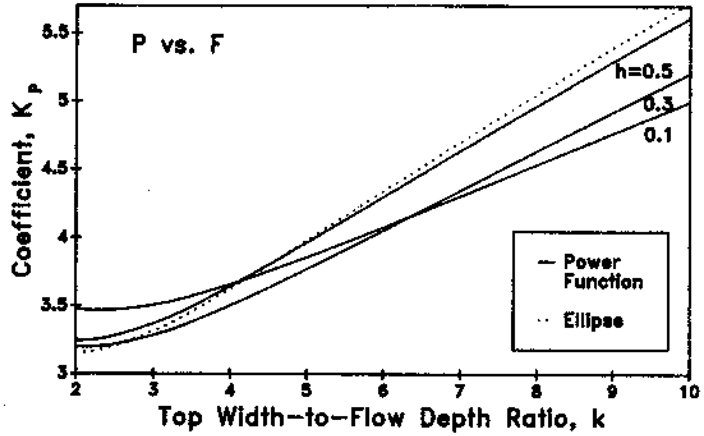

FIG. 13. CS *P* versus *F* Coefficient Variation with *k* and *h*

measured with a profilometer. This time consuming process limits the number of profiles that can be measured. A better procedure is to measure flow depth and top width during irrigation with a rule at a large number of locations and over the relevant range of section factors. Regression of these data (after log transformation) will give b and h values for a composite section.

The RP model perimeter versus depth coefficients, g and u, are generally determined by numerical integration of shapes generated by (3) and regression of the resulting data pairs. A problem with this procedure is that the g and u values are interdependent (negatively correlated) and their determination is sensitive to the specific portion of the perimeter used, especially when h values are small. Although the generated relationship may predict P versus F relationship accurately because of error in the u value and sensitivity of the relative P versus F relationship [(8)] to u. An alternative procedure is to directly measure wetted perimeter with a flexible rule as top widths are being measured and regress the log-transformed P versus D data pairs to determine g and u for a complete section.

With the constant-shape model, the top width-to-flow depth ratio, k, can be easily measured as just described during the irrigation. However, since k is assumed constant, the measurements need not be made over a range of section factors and the measured ratios need only be averaged to determine the best estimate of a representative value for the field. The wetted perimeter-to-flow depth ratio can also be measured if desired. As with k, the average of the measured ratios is the best estimate of k_p. The power-function shape exponent, h, or trapezoid side slope can be estimated, eliminating the need for time-consuming profile measurements. The h value can also be calculated by trial and error from (12) using measured k and k_P values. One-parameter shapes (parabola, ellipse, semicircle) use only k values. Note that the CS geometric coefficients can be determined directly from Figs. 11–13.

Because furrow perimeter shapes change with soil type, soil conditions, and flow time as well as flow conditions, the conditions during coefficient measurement should be similar to those being modeled. For example, if the information is to be used to predict advance trajectories, the data should ideally be collected early in the irrigation during the advance phase. Profile measurement early in an irrigation can be difficult due to unconsolidated perimeter soil and turbid water. If the information is to be used to project final infiltration rate distributions, data should be collected late in the irrigation event during steady flow.

Simplicity

The constant-shape model is simpler to use than the RP model because the exponents of the relationships between the parameters are fixed (independent of shape). As a result, the coefficients are determined more easily and universal relative relationships exist between the geometric and hydraulic parameters [(17)–(21)]. For example, if the flow area is known (or estimated) at one location with a known section factor, the area at any other F value can be easily calculated with no additional shape information. These universal relationships can be used to simplify the complex interactions that occur with surface irrigation. For example, if infiltration is projected to vary with the wetted perimeter then the variation in infiltration rate along a furrow (regardless of shape) can be projected. Likewise, if erosion is related to average shear, then the relative erosion along a furrow can be projected (Kemper et al. 1985). Such analyses provide a better understanding of the complicated interrelationships that affect surface irrigation performance.

Conclusions

The CS model does not fit measured furrow shapes as well as the commonly used RP model and is not conceptually correct during recession. However, it predicts the important relationships between flow area, depth and wetted perimeter, and the channel section factor nearly as well as the RP model and its coefficients are easier to determine. With the CS model, generalized relationships exist between hydraulic and geometric parameters, which gives insight into the processes important to surface irrigation performance.

Furrow shape variability in the field will generally be a more important consideration than the model or shape used. Simple shape measurement procedures that allow measurement at many locations in a field result in more confidence in the important shape coefficients and more representative results.

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Appendix II. Notation

The following symbols are used in this paper:

- A = cross-sectional flow area;
- a = cross-sectional area to some height above furrow bed;
- b = empirical coefficient [(3)];
- c = empirical coefficient (Table 2);
- D = flow depth;
- d = height above furrow bed;
- e = empirical exponent (Table 2);
- err = average absolute deviation between predicted and measured data;
- F = Manning's section factor for uniform flow;
- F_r = relative section factor;
- g = empirical coefficient [(5)];
- h = empirical exponent [(3)];

K_A = coefficient of A versus F relationship [(15)];
K_D = coefficient of D versus F relationship [(14)];
K_P = coefficient of P versus F relationship [(16)];
k = top width-to-flow depth ratio (W/D);
k_a = flow area-to-flow depth squared ratio (A/D^2)
k_p = wetted perimeter-to-flow depth ratio (P/D);
n = Manning roughness coefficient;
P = wetted perimeter;
p = furrow perimeter to some height above furrow bed;
Q = flow rate;
R = hydraulic radius = A/P;
S = furrow slope;
u = empirical exponent [(5)];
W = furrow top width; and
w = furrow width at some height above the furrow bed.