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A New Method for Calculating Frost Heave Including Solute Effects

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A numerical method is presented that models the coupled flow of heat, water, and solutes as unsaturated soil freezes. Input requires a general knowledge of the physical properties of the soil as well as the initial water, temperature, and solute distributions. Soil surface temperature or the heat flux across the soil surface drives the model. The method reproduces the observations of temperature and water movement in two nonsaline field studies previously reported in the literature. The analysis shows that increasing solutes can decrease frost heaving by reducing water flow to the ice lens. A method for measuring the unsaturated hydraulic conductivity in frozen soil is proposed.

INTRODUCTION

Cary and Mayland [1972] noted that the flow of water in frozen soil is similar to the flow of water in unfrozen, unsaturated soil with respect to the liquid phase dynamics. They formulated the equation of state for the energy of the liquid phase in a partly frozen soil system as

$$\gamma = \Upsilon + \Pi \tag{1}$$

where γ is the soil water potential fixed by the vapor pressure of pure ice, Υ is the soil water liquid phase matric potential controlled by phase interfaces, and Π is the osmotic potential due to solutes. They further suggested that the relationship between liquid water content and Υ are nearly the same in both unsaturated unfrozen and unsaturated partly frozen soil. This allowed numerical models of coupled water and heat fluxes to be developed and tested with real data. This approach has gained support [*Nakano et al.*, 1983; *Oliphant et al.*, 1983; *O'Neill and Miller*, 1985; *Guymon et al.*, 1983].

Even though (1) provided the fundamental basis for including solute interactions with the coupling of simultaneous heat and water flow in freezing soils, present numerical models do not include solute effects. *Cary et al.* [1979], using an analysis based on (1) and Darcy-type flow, predicted that "increasing concentrations of solutes oppose frost heave." *Chamberlain* [1983] later verified this and concluded that "the frost-heave susceptibility of sand and clay soils is significantly reduced by saline pore water." (See also *Kay and Groenevelt* [1983].) *Mahar et al.* [1983] further concluded that "Existing numerical methods do not adequately model freezing of saline soils due to the effects of solute exclusion during the phase change."

This paper presents a simple numerical method for estimating the flow of heat, water, and solutes in freezing unsaturated soil. The analysis leads to location and magnitudes of liquid-phase water, ice, temperature, solutes, amount of heave, and heaving pressures as they change with time.

THEORY

The approach follows the method developed by *Cary* [1985] that was used to explain the supercooling of fruit tree blossoms during freezing weather. The basic relations are

$$S = S_0(W/W_s)^{-b} \tag{2}$$

$$k = k_s(W/W_s)^{(2b+2)} \tag{3}$$

$$\phi\Upsilon = S + OP \tag{4}$$

$$J = -k(\Delta S + 1)\Delta tj \tag{5}$$

$$Q = -\mathrm{K}\Delta T\Delta tj + H\Delta W_i \tag{6}$$

$$\delta OP = OPJ - D\Delta OP\Delta t \tag{7}$$

where the symbols are defined in the Notation list at the end of this paper. Equation (2) is an often used empirical relation, (3) was derived from (2) by *Campbell* [1974], and (4) is the form of (1) that was chosen for this application. Equations (5) and (6) are approximate time integrals of Darcy's law and the Fourier heat diffusion equation with a latent heat of fusion term in (6). Mass balance for solutes is given by (7), including terms for molecular diffusion and convective transport.

As unsaturated soil freezes, liquid phase water flows toward the ice crystals. As the crystals grow, they may force the soil particles apart and become a continuous layer of nearly pure ice. The passage of solutes and liquid-phase water through this layer ceases, and additional terms must be included in its equation of state (equation (4)), because the film of liquid phase water adsorbed at the ice interface must support the soil overburden and any load on the surface. Thus

$$\phi T = S + OP + HP \tag{8}$$

where HP, the heaving pressure, is the force per unit area arising from the overburden on the ice layer. The temperature may not be low enough at the time total saturation occurs to generate a heaving pressure large enough to equal the sum of the overburden plus a surface load. Therefore liquid will not flow into this region until the water suction increases enough to overcome the overburden and any surface load. During this initial period, static mechanics suggest that

$$HP = S \tag{9}$$

As the suction increases, the liquid phase will resume flow into the saturated area next to the ice layer, separating the soil particles so that

$$HP = \text{overburden} + \text{surface load} \tag{10}$$

$$S > HP \tag{11}$$

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so long as the thickness of the ice layer is increasing. Substituting (9) into (8) gives the maximum heaving pressure that can be developed as

$$HP_{max} = 0.5(\phi T - OP) \tag{12}$$

Equation (12) shows that the heaving pressure is always less than the theoretical thermodynamic maximum (approximately $-\phi T$) and that heaving decreases as the amount of solutes increases. Equations (8) through (11) provide the additional information, beyond (2)–(7), to form a numerical description of the behavior of soil heaving as it freezes.

The method is based on volume elements of soil starting at the surface and going as deep as needed to reach a temperature and water content that remains relatively constant. It is assumed that the physical characteristics of the soil are known (see the constants in (2), (3), (5), and (6)), as well as the initial distribution of temperature, water content, and solutes. The soil surface temperature or the heat flux across the soil surface must be known or assumed during the freezing period to drive the model.

First, the heat flux into and out of the surface volume element is calculated over the time increment Δt with (6). A similar calculation using (5) follows for the water flux. Then the solute transfer is estimated with (7). Mass and energy conservation give updated values for temperature, water, and solute levels. An updated soil suction follows from (4), and a current liquid water content comes from (2). The amount of ice is the difference between liquid water content and the total water which is always known from the calculated values of J and the mass balance. Latent heat follows from changes in ice content during each time increment.

Since the heat flux is toward the surface as a soil freezes under natural conditions, the latent heat release must not exceed the heat lost through the surface by thermal conduction. Therefore a subroutine is included to check and correct the water flux so that the Fourier and fusion heats remain correctly coupled. Similarly, the osmotic pressure cannot exceed the value of ϕT in (4), so a subroutine is also used to check this coupling and correct the solute flux if necessary.

When saturation is reached in any volume element, (8) replaces (4), and no more water is allowed to flow into that element until its suction exceeds the sum of the overburden, the surface load, and the suction in the element directly below. As the liquid plus ice content at any depth increases beyond the initial soil pore volume fraction, the difference is the amount of heave. In (5) through (7) the length parameter j, which here was 1 cm since cubic centimeter volume elements were used, was increased accordingly as the pore volume increased with heaving. Changes in water density as ice forms, vapor phase latent heat, vapor diffusion, and hydrodynamic dispersion were not included in the algorithm.

The program, approximately 120 lines written in BASIC, is available from the author with a line by line explanation of the logic.

RESULTS AND DISCUSSION

Simulation of Field Data

The simulation of field observations reported by *Cary et al.* [1979, Figure 5] is presented in Figure 1. The soil water content and temperature were measured on a field plot of silt loam at 4:00 P.M. and again the next morning at 8:00 A.M.

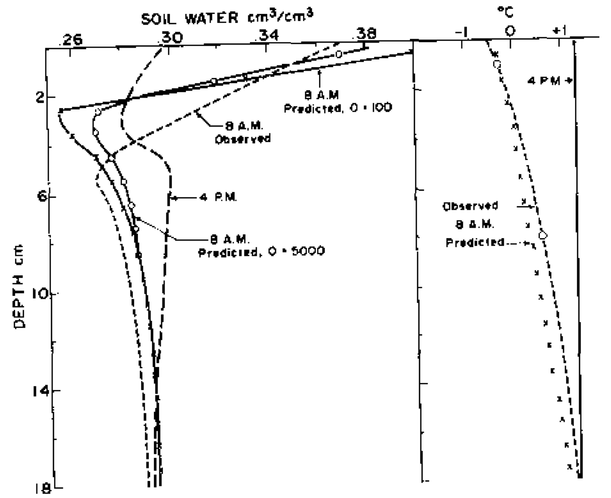

Fig. 1. Observed and predicted water and tempertaure profiles following an overnight freezing event. The model's predictions are made at 1-cm-depth increments and are shown as points. The parameter OP gives the initial osmotic pressure (centimeter of H_2O) in the soil solution.

after a light freeze. The data show rapid flow of water to the soil surface that is typical of a slowly freezing, wet soil. Simulation of the data with the model described here is satisfactory. It was assumed that $b = 6$, which characterizes the soil's unfrozen water relations and also gave the best fit between the model and the observed data. The 5000 cm of water curve predicts the change if the soil had been moderately saline. For this simulation the soil surface temperature was lowered by the freezing point depression of the osmotic solution, so the result is comparable to the 100-cm H_2O osmotic pressure curve. The osmotic pressure of 100 cm of H_2O is a very low solute level. Figure 2 compares the model's simulation to field data for the same soil over the same time period as that shown in Figure 1, except that the initial water content was much less. Increasing the osmotic pressure made no significant difference in the model's prediction in this case.

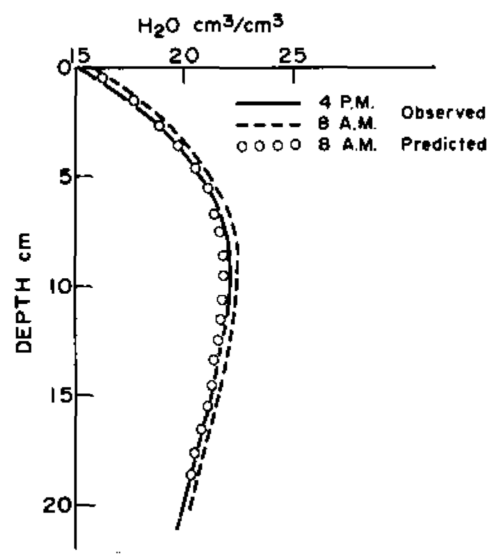

Fig. 2. Predicted points at 1-cm increments and observed curves showing the effect of low initial water content on soil water movement during the same freezing event occurring in Figure 1.

Pikul and Allmaras [1985] have also reported some field observations of soil freezing overnight (Figure 3). The model simulated their observations of temperature change, water movement, and depth of freezing quite well. A "*b*" value of 5 and an osmotic pressure of 100 cm H_2O were reasonable values for the unfrozen soil and also produced a good simulation.

Heave Sensitivity and Soil Structure Changes

The predicted effects of surface load and solutes are shown in Figure 4 for the same silt loam as Figure 3, except that the initial water content was 45%. The freezing period was 16 hours, beginning at 0°C with the surface temperature falling to −4°C during the first 8 hours and remaining there for the next 8 hours. Liquid was allowed to flow across the lower boundary under the constant initial conditions at the 20-cm depth. Water contents greater than 0.58 represent ice layering and soil heaving. The formation of ice is a sink for water that increases the pressure gradients in the surrounding liquid phase. Since solutes reduce the rate of freezing, it follows that they also reduce heave and the upward flow of water. The surface load decreased ice at the surface but let a second ice layer form and heave at 5 cm. The simulated accumulation of solutes near the surface is shown in Figure 5 as relative concentrations, i.e., the amount of solute after 16 hours of freezing divided by the initial amount in each cubic centimeter of depth increment. A significant flux of water and solutes upward across the 20-cm lower boundary occurred.

A longer simulation for the soil in Figure 1 is shown in Figure 6 with the initial water content at 0.35. The surface temperature was −0.5°C in the low osmotic pressure case. For the moderately saline case ($OP = 5000$) the surface was held at −0.9°C. This lower surface temperature compensated for the lower freezing point to make the freezing phenomena comparable. There was no heaving in the saline soil, and the ice penetration was deeper. In both cases, flow was allowed across the 20-cm lower boundary where all initial soil parameters were assumed to be constant.

Freezing and thawing affects soil structure. The water content (liquid plus ice) must exceed the initial pore space to significantly change the particle arrangement in aggregates or layers of soil [*Bullock*, 1986]. The conditions that cause soil structure changes are therefore closely coupled to heaving. The layers where soil structure will change are obvious in Figures 4 through 6, i.e., where the water content exceeds the initial pore space. Soil structure changes due to freezing initially lead to a loss of cohesion between individual particles [*Formanek et al.*, 1984], but as the ice pushes the particles apart, pore space increases. In compacted clods or layers this leads to a more favorable aggregate structure, if time is allowed for particle to particle bonding to re-form after thawing and drainage of excess soil water [*Kemper and Rosenau*, 1984]. On the other hand, compaction or erosion soon after thawing, but before the particle contact points have had time

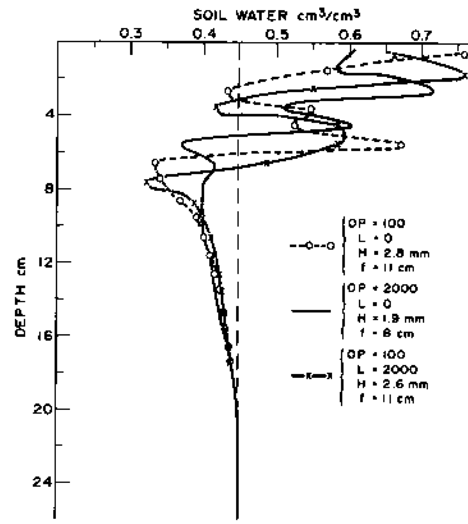

Fig. 4. Predicted change in water distribution during 16 hours of freezing as affected by the parameter values shown. The initial water content was 0.45; *OP* was the initial osmotic pressure and *L* the surface load (both in centimeters of H_2O), *H* was the amount of soil heave, and *f* was the depth of ice penetration.

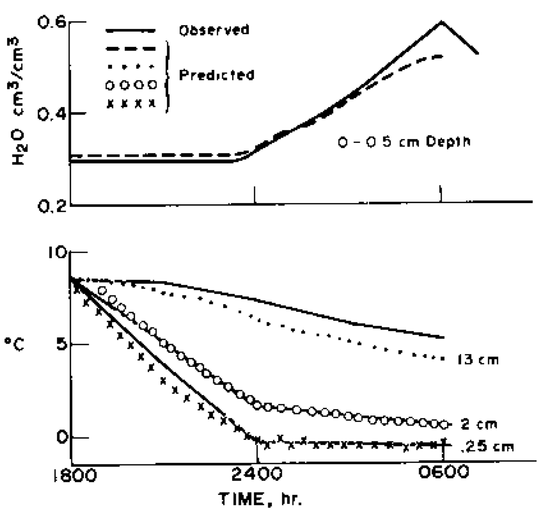

Fig. 3. Observed and predicted soil surface water content and soil temperature at 0.25-, 2-, and 13-cm depths during an overnight freezing event.

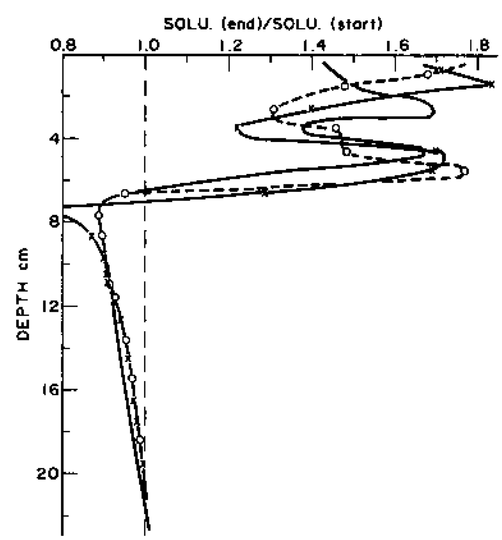

Fig. 5. Predicted redistribution of solutes for the same conditions prevailing in Figure 4. The solutes are expressed as the ratio of the mass of solute at the start to the mass at the end of the 16-hour freezing period.

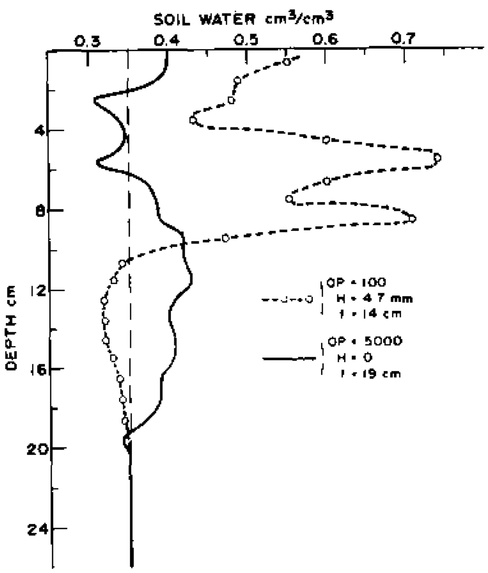

Fig. 6. Predicted redistribution of water during a 9-day freezing period, starting with a uniform level of 0.35 cm^3/cm^3. The curve parameter identifications are the same as in Figure 4.

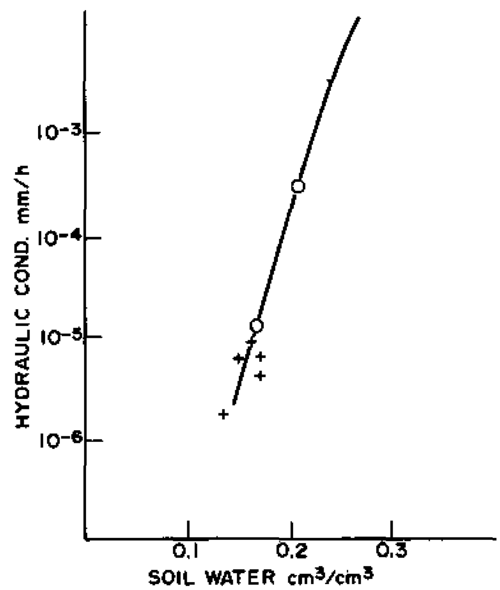

Fig. 7. The hydraulic conductivity of a silt loam soil for unfrozen conditions is compared to conductivity of the liquid phase in frozen soil. The solid line and open circles are the lower portion of a curve previously published by *Robbins* [1977] for the unfrozen soil, while the crosses were calculated from the data of *Cary and Mayland* [1972] for the frozen soil.

to recement, will be detrimental from watershed and agricultural standpoints.

Well-instrumented observations of soil freezing under a wide variety of conditions in the field are needed to test the validity of this numerical approach. Under very wet conditions and in weather that promotes rapid freezing, numerous thin layers of ice may form near the soil surface [*O'Neill and Miller*, 1985]. Smaller-depth increments requiring more computer time would, of course, have to be used in the algorithm proposed here to reproduce that type of result.

Frozen-Unfrozen Similarity Question

The assumption that the functional relationship between liquid water content and hydraulic conductivity or the liquid content and suction are the same in frozen and unfrozen soils requires additional scrutiny. These two systems differ by the presence or absence of the liquid-ice interface. An advantage of the method proposed here is that the problem is approached through the value of the pore size distribution factor b [*Campbell*, 1974]. It is chosen to give optimum agreement between the model and experimental observations. The unanswered question is whether or not values of b obtained from unfrozen measurements may always be used without modification in the freezing system. It is possible that future experimental data may lead to expressing b as a function of ice content. For the present, however, the limited measurements in the field suggest that values of b from unfrozen soils do a fairly good job of describing the freezing system (Figures 1 through 3).

Direct measurements of the unsaturated hydraulic conductivity and suctions in frozen soil are of considerable interest, but they are difficult to obtain with confidence [*Horiguchi and Miller*, 1983; *Nakano et al.*, 1983]. Figure 7 shows five observations of the hydraulic conductivity in a frozen column of Portneuf silt loam. They are compared to the hydraulic conductivity of the same soil in the unfrozen condition. The agreement is good, probably fortuitously so in light of the uncertainties in such measurements in both the frozen and unfrozen states. The unfrozen conductivity curve was published by *Robbins* [1977], based on experimental measurements and the same capillary pore size distribution theory that is the basis for the constant b used here. The frozen conductivity values were obtained from the data of *Cary and Mayland* [1972, Figures 2*a*, 4, and 5]. The area between their steady state "3-week and 9-week" frozen water content curves was graphically determined, and the net water fluxes across five sections of the column were divided by the respective hydraulic gradients, based on (1) and shown graphically in the Cary and Mayland paper.

This method, using observations of steady state freezing water flow, is the most direct experimental technique presently available for measuring the hydraulic conductivity as a function of liquid water contents in frozen soil. One must be sure that the water distribution is known when the temperature field becomes constant, since a relatively large amount of liquid flow occurs during the initial freezing period before the temperature reaches a steady state distribution. Perhaps moist soil can be "quick frozen" and then warmed to the desired freezing temperature to reduce this problem. It is also important that the solute concentration be known and relatively constant. Wetting the soil with a saturated solution of potassium sulfate is, because of its intermediate solubility, a convenient way to buffer the osmotic pressure of the liquid phase in this type of experimental approach.

NOTATION

- b pore size distribution characteristic [*Campbell*, 1974].
- D effective diffusion coefficient of solutes, hour^{-1}.
- H latent heat of fusion, cal/cm^3.
- HP heaving pressure, cm H_2O.
- j depth increment of each volume element, cm.
- J volume flow of water, cm^3/cm^3.
- k hydraulic conductivity, cm/h.
- K thermal conductivity, cal/(cm h °C).

OP solute concentration expresed as osmotic pressure, cm H_2O.

δOP solute transported, cm H_2O.

Q heat transported, cal/cm^3.

s subscript indicating saturation.

S soil water liquid-phase suction, i.e., the apparent negative pressure with respect to ambient pressure, cm H_2O.

S_0 air entry suction, cm H_2O.

t time, hours.

T temperature, °C.

W liquid phase concentration, cm^3/cm^3.

W_i ice phase concentration, cm^3/cm^3.

W_s soil pore volume fraction, cm^3/cm^3.

Δ change per centimeter or difference per time increment.

ϕ factor to convert temperature to apparent water suction in a freezing soil (approximately $-12{,}000$ cm H_2O/°C), i.e., the pressure equivalent to the vapor pressure difference between pure ice and liquid water at any given tempertaure in the range of zero to a few degrees below freezing [*Cary and Mayland*, 1972].

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