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# AMOUNT OF SOIL ICE PREDICTED FROM WEATHER OBSERVATIONS*

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ABSTRACT

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An equation has been developed that gives the net daily heat flux across the soil surface in the winter. The equation is based on the Fourier heat flow relation using the thermal gradients and conductivity at the soil surface. Input requires the daily maximum/minimum air temperatures, solar radiation and snow depth. The cumulative daily soil heat flux was used to estimate the amount of ice in the soil. Several years of data from weather stations near Lafayette, Indiana, and Twin Falls, Idaho, were used to test this approach. Given a site constant that accounts for soil type and cover conditions, it appears that the presence or absence of soil ice can generally be correctly predicted at least 70% of the time over a period of years that includes both warm and cold extremes.

## INTRODUCTION

Frozen soil reduces the infiltration of water. This is an important factor in the hydrology of watersheds that experience low temperatures. The freezing and thawing of soil depends on meteorological variables such as air temperature, radiation, wind and humidity as well as slope, aspect, soil properties and cover. In many practical cases, the only information one may have is the daily maximum/minimum air temperatures, precipitation, sometimes solar radiation and a general description of various sites with respect to slope, aspect, cover and soil type. The ability to predict the occurrence of frozen soil from this type of information would be quite useful in hydrologic modeling of water storage and runoff. If the daily maximum soil temperature at any depth is 0°C or less, it is reasonable to assume that the soil contains ice. Consequently, models that predict soil temperatures from daily maximum/minimum air temperatures (Bonham and Fye, 1970; Hasfurther and Burman, 1974; Toy et al., 1978; Parton and Logan, 1981) are potentially useful for predicting the occurrence of soil ice, but all these methods at present have some drawbacks. An alternative approach is to calculate the daily net soil heat flux and infer the formation or melting of ice from the sum of these values. This concept was tested in a one-season

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study, by Cary et al. (1978). They found a simple heat flow relation, using soil thermal gradients across the surface based on daily mean air temperatures, gave better predictions than the total energy balance equation using maximum/minimum air temperatures and solar radiation for inputs. The purpose of the study reported here was: (1) to develop a more fundamental soil heat flow relation to utilize maximum/minimum air temperatures and solar radiation as inputs; and (2) to test soil ice prediction capabilities over a period of years having different weather patterns.

## THEORY

The soil temperature $T$, may be approximately expressed as a function of depth $Z$ at any time $t$, as

$$T = T_a + A_0(\exp - Z/d) \sin(\omega t - Z/d) \tag{1}$$

where $T_a$ is the average soil temperature (°C), $A_0$ is the amplitude of $T$ at the surface, $\omega$ is the frequency and $d$ is the damping depth (van Wijk and deVries, 1963). The heat flux $F$ across the soil surface may be written as

$$F = \mathrm{d}G/\mathrm{d}t = -\lambda(\mathrm{d}T/\mathrm{d}Z) \tag{2}$$

where $G$ is the amount of heat per unit area and $\lambda$ is the thermal conductivity (cal °C$^{-1}$ h$^{-1}$ cm$^{-1}$). Equation 1 may be differentiated to derive the temperature gradient and used in eq. 2 to solve for net heat flux, leading to

$$G_n = (\lambda_w A_w)/d_w \int_{t_1}^{t_2} (\sin \omega_w t + \cos \omega_w t)\, dt + (\lambda_c A_c)/d_c \int_{t_2}^{t_3} (\sin \omega_c t + \cos \omega_c t)\, dt \tag{3}$$

where $G_n$ is the net heat flux across the soil surface during a 24-hour period and the subscripts w and c refer to the warming and cooling cycles, respectively. The form of eq. 3 is based on the assumption that the warming and cooling cycles may be described by sine waves with different periods; i.e., by brief warming and a longer cooling period due to short days in the winter. Integrating eq. 3, and choosing limits to give $\omega t_1 = 3\Pi/2$, $\omega t_2 = \Pi/2$ and $\omega t_3 = 3\Pi/2$, gives

$$G_n = [(2\lambda_w A_w)/(d_w \omega_w)] - [(2\lambda_c A_c)/(d_c \omega_c)] \tag{4}$$

or

$$G_n = \lambda/\Pi \left[\frac{(T_h - T_{lw})t_w}{d_w} - \frac{(T_h - T_{lc})t_c}{d_c}\right] \tag{5}$$

where $T_{lw}$ is the soil surface low temperature that occurs early in the morning, $T_h$ is the soil surface high temperature that comes later in the day



and $T_{lc}$ is the soil surface low temperature that occurs the following morning. The soil thermal conductivity, $\lambda$, is assumed to be constant over the 24-hour period in which $t_w$ represents the hours of warming and $t_c$ the hours of cooling. Equation 5 is of special interest because it illustrates the large effects soil freezing and thawing have on heat flow across the surface since both the damping depth and thermal conductivity change significantly with the formation of ice.

When the top 0.5—1 m of soil is near 0°C but contains no ice, the 'soil heat' $P$, is taken as zero. Stating with this day, $J_0$, subsequent values of $P$ are defined as

$$P = \sum_{J_0}^{J} G_n + G_u \tag{6}$$

where $J$ is the Julian date and $G_u$ is the daily heat per unit area that flows upwards into the surface layers of soil in the winter. When $P$ is positive, the soil is not frozen, while negative values indicate the approximate amount of ice in the soil. Given daily values of maximum/minimum air temperatures and estimates of snow depth and solar radiation, one must then develop functions that use these weather station observations to give values for the variables in eq. 5 so that $G_n$ may be calculated.

## METHODS

A field study similar to the one described by Cary et al. (1978) was conducted near Twin Falls, Idaho, during the winter of 1977—1978 using four instrumented sites near the Kimberly weather station. The sites were: (1) a grass-covered 15° north slope; (2) a bare 25° south slope; (3) a level bare soil; (4) a level grass-covered soil surface. Some of the snow that fell during the winter was artificially managed on the level area to provide soil temperature data affected by snow depth. Results from this study and particularly the detailed data base developed during the work at Pullman, Washington, reported by Cary et al. (1978) were used to develop empirical relations between available weather station measurements and the variables in eq. 5. These relations (eqs. 7—10) were developed by curve fitting, intuition based on experience, and a large number of sensitivity trials on the computer.

The damping depth was approximated by $d = 15\, T_a/(T_a - 1)$ based on the work of Fuchs et al. (1978) where $d$ is in cm and $T_a$ is an average soil temperature where ice is forming. Values of $T_a$ were taken as

$$T_a = -0.13\, \bar{T}^2 + 0.2\, \bar{T} - 0.8 \tag{7}$$

where $\bar{T}$ is a weighted mean soil surface temperature for either the warming or cooling cycle. During the warming period, $\bar{T} = 0.7\ (T_h - T_{lw}) + T_{lw}$ and for the cooling period, $\bar{T} = 0.3(T_h - T_{lc}) + T_{lc}$. To insure stability, values of $d$ less than 5 are taken as 5. When there is no ice in the soil, $P \geqslant 0$ and $d = 15$. Values of $T_h$ were taken as the air temperature maximum plus

'$C$', which is a factor accounting for the effects of solar radiation on soil temperature. Values of $T_{lw}$ and $T_{lc}$ were taken as their respective daily low air temperatures plus $0.5\,C$. The weighting factors 0.7, 0.3 and 0.5 in these temperature relations were included to help account for the skewed-type sine function of temperature that often occurs in the winter.

Potential solar radiation $S_{t0}$, was taken from the tables by List (1951). Measured values of solar radiation $S_t$, were then used in the relation

$$C = D(S_t/S_{t0})^{1/2} \quad (8)$$

where $D$ is an empirical constant specific for each site. The effects of soil, cover, and other physical properties associated with individual sites are accounted for by the value chosen for $D$. When the site is on a slope the value of $C$ given by eq. 8 is multiplied by $S_t'/S_{t0}$ where $S_t'$ is the daily solar radiation corrected for slope and aspect with the tables given by Buffo et al. (1972).

The maximum/minimum soil surface temperatures ($T_h$, $T_{lc}$ and $T_{lw}$) were corrected for damping by snow cover. When either the maximum or minimum air temperatures were zero or greater, the respective maximum or minimum soil surface temperature was taken as zero under snow. On the other hand, when either the maximum or minimum air temperatures were negative, the soil surface temperature was taken as $5T/(I + 5)$ rather than $T + C$ or $T + C/2$, where $I$ is the snow depth in cm and $T$ is the appropriate daily maximum or minimum air temperature.

The soil warming period $t_w$, is given by

$$t_w = C - 1 + 2\cos(J/57.3) + Y + t_0 \quad (9)$$

where $t_0$ is the hours of daylight on day $J$ calculated from Campbell (1977). The factor $Y$ depends on where the maximum/minimum air temperatures fall with respect to 0°C; normally when $T_h \geqslant 0$, $Y = 0$, but when $T_h < 0$, $Y = -[1 + \ln(1 - T_h)^{1/2}]$ or if $T_{lw} > 0$, $Y = [\ln(T_h + T_{lw})]^{1/2}$ when $P < 0$. These relations help correct for the more rapid freezing or thawing that occurs when the air temperature does not rise above or fall below 0°C during the 24-hour period.

Two additional controls were used for stability. When $P > 0$, it is not allowed to go negative unless $(T_h + T_{lw} + T_{lc})/3 < 0$, and when $P$ goes from $< 0$ to $> 0$ its maximum value at the end of that particular 24-hour period is limited to $10\,\text{cal cm}^{-2}$. The first of these conditions prevents the soil from entering the frozen state unless the average air temperature is below freezing for 24 hours. The second condition takes account of the abrupt increase in damping depth when the soil thaws, causing a rapid reduction in the heat flux across the surface. It is assumed that $\lambda = 7.8\,\text{cal h}^{-1}\,\text{cm}^{-1}\,°\text{C}^{-1}$ for unfrozen soil and 17.9 for frozen soil.

Values for upward heat flux were taken as

$$G_u = (A - 0.2T_a)\sin[(J + 80)/57.3] \quad (10)$$

where $A$ is a constant that characterizes the magnitude of upward heat flux from the subsoil during the winter period.

Records from weather stations at Kimberly, near Twin Falls, Idaho, and Lafayette, Indiana, were used to test these relations in eq. 5. The simple equation previously developed by Cary et al. (1978) was also tested with the weather records from Kimberly and Lafayette. It was used in the form

$$G_n = 15\{\bar{T}_a - (\bar{T}_{a-1}/B)[5/(5+I)]\} \quad (11)$$

with

$$G_u = A + [1 + 0.2(|\bar{T}_{a-1}|)^{1/2}]\sin[(J+80)/57.3] \quad (12)$$

where $\bar{T}_a$ is the average air temperature on day $J$ and $\bar{T}_{a-1}$ is the previous day's average. Additional program controls were imposed on eqs. 11 and 12 so that $P$ could not be greater than zero, and if $P = 0$ on day $J = 1$, then $G_n = 0$ on day $J$ unless $\bar{T} < -3$. Thus the soil will not begin a continuously frozen period on any day that the average temperature is not at least $-3°$C.

The predictions were made with a single specific site constant, $D$ (eq. 5) or $B$ (eq. 11) that was not allowed to change from year to year. For Twin Falls, $D = 0.5$, $B = 1.2$ (grass surface) and for Lafayette, $D = 1.1$, $B = 0.9$ (grass surface), or $D = 0.5$ and $B = 1.3$ (bare surface). The constant $A$ was assigned values of 1.5 for Twin Falls and Lafayette, and 2.8 for Pullman. These specific site constants were found by trial and error using the weather records that included soil temperatures.

The calculations were begun in December as the soil temperatures approached freezing, indicating that the soil heat, $P$, was approaching zero. At Lafayette, the bare soil reached this temperature before the grass-covered area. Consequently for some years, depending on the soil temperatures given in the weather records, the calculation was begun with $P$ having positive values between zero and 100 cal $cm^{-2}$ for the grass cover. Initial values of $P$ may be estimated from the average morning temperature of the top 30 cm of soil multiplied by a soil heat capacity of 0.5 cal $cm^{-3}$ $°C^{-1}$ and a depth of 30 cm. When soil temperatures are not available from the weather records, a brief field survey using a soil temperature probe should be adequate to find the initial value of $P$ that is needed to start the calculations each winter.

## RESULTS AND DISCUSSION

The results are summarized in Table I as the percent of days during the soil freezing season that eqs. 5 and 11 correctly predicted the persistence of ice in the soil over the entire 24-hour period. The weather station records of daily maximum/minimum soil temperatures were used to judge the presence of soil ice, i.e., if maxima at any of the depths recorded were not above 0°C, it was assumed that the soil had remained frozen over the 24-hour period. The years analyzed were specifically chosen to cover a range of air temperature and snow cover patterns, for the objective was to find how well the use of a single site constant would predict frozen soil conditions from year to year.

TABLE I

The accuracy of soil ice predictions made from eqs. 5 and 11

| Site location and description | Year | Equation 5 (% of days correctly predicted) | Equation 11 | No. of days with soil ice | No. of days in the freezing season |
|---|---|---|---|---|---|
| Lafayette, IN | | | | | |
| Grass | 1972—1973 | 74 | 55 | 22 | 89 |
| Bare | | 70 | 83 | 77 | |
| Grass | 1973—1974 | 95 | 62 | 0 | 84 |
| Bare | | 77 | 75 | 14 | |
| Grass | 1974—1975 | 97 | 66 | 0 | 106 |
| Bare | | 68 | 74 | 54 | |
| Grass | 1975—1976 | 68 | 54 | 28 | 72 |
| Bare | | 70 | 99 | 60 | |
| Grass | 1976—1977 | 88 | 50 | 62 | 94 |
| Bare | | 85 | 82 | 75 | |
| Grass | 1977—1978 | 90 | 46 | 76 | 119 |
| Bare | | 81 | 85 | 105 | |
| | Std. Dev. | 11 | 16 | | |
| Kimberly, ID | | | | | |
| Grass | 1974—1975 | 75 | 86 | 65 | 99 |
| Grass | 1975—1976 | 25 | 65 | 66 | 73 |
| Grass | 1978—1979 | 65 | 93 | 73 | 95 |
| Grass | 1980—1981 | 67 | 54 | 11 | 71 |
| Grass | 1977—1978 | 87 | 75 | 12 | 95 |
| Bare | | 88 | 72 | 13 | |
| Grass, 15° N slope | | 79 | 75 | 18 | |
| Bare, 25° S slope | | 90 | 93 | 3 | |
| | Std. Dev. | 21 | 14 | | |
| Pullman, WA | | | | | |
| Grass | 1976—1977 | 89 | 85 | 51 | 61 |
| Grass, 25.5° N slope | | 93 | 79 | 58 | |
| Grass, 18.5° S slope | | 77 | 67 | 29 | |
| Bare, 11.5° S slope | | 89 | 74 | 44 | |
| Straw, 11.5° S slope | | 89 | 84 | 53 | |
| | Std. Dev. | 6 | 7 | | |

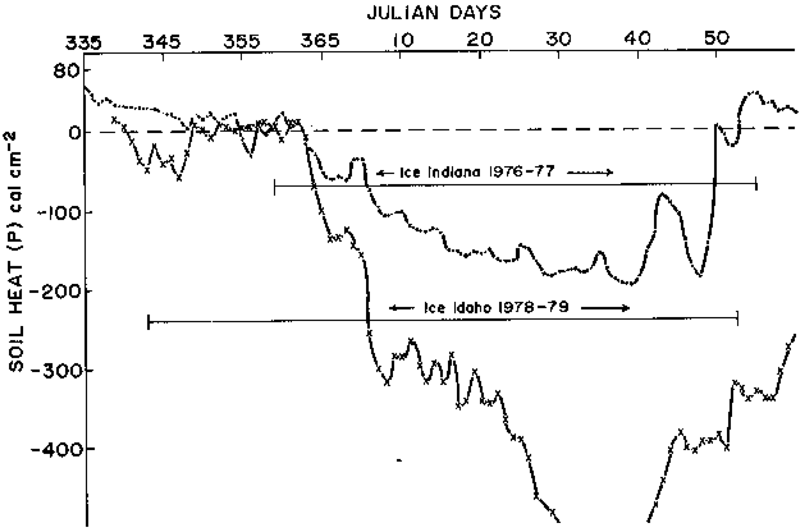


Fig. 1. Prediction of soil heat from eq. 5 as a function of time for two different winters at Lafayette, IN, and Twin Falls, ID. The presence of soil ice, based on soil temperature measurements is shown by the solid lines with brackets at each end. Negative values of soil heat predict the amount of soil ice while positive values indicate its absence.

Four seasons of soil heat calculations are shown in Figs. 1 and 2. Soil ice predictions for the bare soil in Lafayette, Indiana, were fairly good in 1976—1977. Predictions for the other three seasons shown were among the poorest of all the cases studied. The possibility of errors in detecting the presence of soil ice from 0°C maximum soil temperatures given by weather station records, must be recognized. Thermometer errors of just a few tenths of a degree could lead to the wrong conclusions, particularly as soil temperature is well buffered between 0 and $-0.5$°C during freezing and thawing processes. This is, of course, why the soil temperature models mentioned in the introduction cannot be expected to be better indicators of soil ice than results given here, for at best, their accuracies are 1 or 2 degrees. However, the air temperature to soil temperature convolution model developed by Hasfurther and Burman (1974), while requiring a more difficult data fitting procedure, might be modified to find the soil surface temperatures needed in eq. 5 and so lead to improved accuracy in soil heat predictions. On the other hand, note that the average accuracy of the very simple eq. 11 (Table I) is 73%, while that of the more fundamental eq. 5 is 79%; an improvement of only 6% which may not really be significant, judging from the standard deviations given in Table I. Either method may be about as well as one can do given only daily maximum/minimum air temperatures with estimates of cloud cover and snow depth. The accuracy of predicting soil ice may be limited more by the lack of uniqueness between soil heat flux and the daily air temperature extremes than by the form of the function

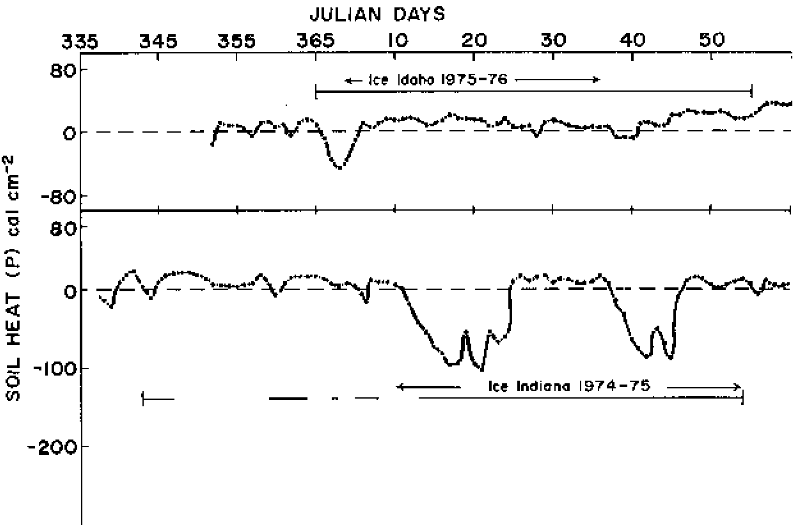


Fig. 2. The same type of results shown in Fig. 1, but for different years. These soil 'heat' curves and the Idaho curve in Fig. 1 gave some of the poorest predictions of soil ice during the course of the study, as may be seen from Table I.

used to find $P$. See, for example, the limited success of Gupta et al. (1981) in predicting hourly soil surface temperatures from known hourly values of air temperature.

## CONCLUSIONS

The method developed here (eq. 5), will describe the presence of soil ice quite well if the coefficient $D$ is adjusted for each individual year. This is, of course, not practical when the objective is to infer the occurrence of soil ice associated with past weather records. On the other hand, if one wishes to predict the data of soil thawing based on current conditions and weather forecasts, it may be practical to make some adjustments in the site constant so that the current season's soil ice predictions agree closely with weather station soil thermometer readings.

At present, the site constant $D$ must be experimentally determined for various soil and cover conditions. This can be done by matching soil ice predictions to soil temperature measurements over the course of one or more winters. The constant can then be used for other slopes and aspects, provided cover and soil properties do not change. This may be the principal advantage of eq. 5 with respect to eq. 11. Generally, it appears that predictions on the average over a number of years will be at least 70% accurate, even when the same site constants are used every year.

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