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Is the Soil Frozen or Not? An Algorithm Using Weather Records

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Frozen soil water is important in hydrologic events because it reduces water infiltration. The presence of soil ice can be predicted reasonably well from detailed knowledge of the soil and microclimatic variables, but this type of information is generally unavailable. Consequently, the purpose of this study was to start with fundamental relations and see how well frozen soil conditions could be identified from daily weather station records of maximum-minimum temperatures, solar radiation, and snowfall. Two relations were developed, one based on the soil-atmosphere energy budget and the other on the heat flux across the soil surface layer. Conceptually, the two equations may be used together to give daily snowmelt as well as soil thawing and freezing rates, but in practice, the snowmelt prediction is probably not yet accurate enough for most practical applications. The simpler equation, describing the heat flux in the soil surface, does not require solar radiation input, yet it gave fair predictions of frozen soil on five diverse sites studied in the Palouse region of eastern Washington. Both approaches require only a single constant that accounts for individual site conditions such as slope, aspect, cover, and soil properties.

Introduction

Soil ice reduces the infiltration of water. Consequently, it is an important factor in predicting floods, seasonal water supplies, and soil erosion from runoff. If detailed information on soil properties and microclimate is available, reasonably good estimates of soil freezing and thawing can be made [*Dempsey and Thompson*, 1969]. More often, however, one has only a general description of the site and records of daily precipitation and maximum-minimum temperatures from a regional weather station. Occasionally, daily values of solar radiation are also available. Consequently, the objective of this study was to develop, from fundamental considerations, methods for predicting when the soil was frozen using daily weather station records of temperature, solar radiation, and precipitation.

Methods

Five sites, each representing a different slope or aspect of the diverse Palouse topography in the Pacific Northwest, were selected for study. Characteristics of these sites, located on the Palouse Conservation Field Station near Pullman in eastern Washington, are listed in Table 1. The soil was Palouse silt loam or a closely related series (Pachic Ultic Haploxerolls), formed from loess with deep, permeable profiles. At the onset of freezing weather in December the soil water content was near field capacity in the surface 20–30 cm and dry to the plant-wilting point below this level to a depth of 1.5 m.

Thermocouples and heat flux meters were installed on all the sites, and measurements were made either automatically at 2-hour intervals or manually during periods of interest. Frost tubes similar to those described by *Rickard and Brown* [1972] were installed on all sites. (The frost tubes were modified near the end of the experiment with a small cotton thread that passed from the fluid in the tube past a stopper in the cap to the atmosphere. This initiated ice nucleation and reduced supercooling of the fluid. Thin-walled rubber tubing was imbedded in sand in the center of the tube and vented at the bottom to absorb the volume increases from freezing.) Maximum and minimum air temperatures and precipitation were measured at a weather station 1 km from the sites. Solar radiation was measured on the Washington State University campus 10 km distant. All data were obtained with standard commercial instruments, except the soil heat flux meters, which are described by *Fuchs and Hadas* [1973].

The winter climate of eastern Washington is humid with mixed rain-snow precipitation and interspersed periods of freezing-thawing weather. Frost depth is usually less than 30 cm, but soil may freeze and then completely thaw several times during the winter, thaw often being accompanied by rain or melting snow.

Theory

Concepts

Analysis of the soil freezing and thawing problem can be simplified by considering only the net daily heat flow across the soil surface. When the top 30 cm or so of soil has cooled to near 0°C, almost all further soil heat loss comes from freezing water because the latent heat of freezing is much greater than the heat capacity of soil. Thus at this temperature the daily heat flow into or out of the soil may be interpreted as freezing or thawing of water. When the sum of daily heat flows is negative, ice must be present in the soil, while when it is positive, the soil is unfrozen. This concept may be concisely stated as

$$M = \sum_{n=1}^{n} (G_n + \mathrm{up}_n) \qquad (1)$$

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TABLE 1. Description of Study Sites and Values of Constants Used to Calculate Frost Conditions

Site			Values of Constants			
Ground Cover	Aspect	Slope, deg	h	A	B	k/l
Bare	south	11.5	1.6	0.75	1.00	20
Wheat stubble	south	11.5	1.6	0.73	1.10	15
30% short grass	south	18.5	1.9	0.75	0.90	20
60% short grass	east	8.0	$K = 1$	0.90	1.10	15
50% short grass	north	25.5	0.0	1.16	1.15	15

where $M < 0$ indicates soil frozen and $M \geq 0$ indicates soil not frozen. In this relation, n indicates the day beginning with the soil near 0°C, G_n is the daily average soil heat flux downward across the surface, and up_n the daily soil heat flux upward from subsoil layers into the zone susceptible to freezing. The problem thus becomes one of evaluating (1) from weather station records of daily temperature extremes, solar radiation, and snowfall.

Values of up_n determined from measurements of thermal conductivity and soil temperature gradients were relatively small, of the order of 2 W m^{-2}. For this study the empirical relation

$$\text{up}_n = 2.5 \sin (J + 80) \tag{2}$$

where J is the Julian date, gave values of mean daily upward soil heat flux that followed the experimental measurements reasonably well.

The downward soil heat flux G may be estimated in two ways: (1) from the net energy exchange between the soil surface and the atmosphere and (2) from the heat conducted across the soil surface.

The daily heat conducted across the soil surface is approximately

$$G_n = (k/l)(T_s - T_0) \tag{3}$$

where T_s is the average soil surface temperature, T_0 is the average soil temperature at some shallow depth l, and k is the average soil thermal conductivity over l.

Soil heat flux from the energy balance approach is given by

$$G = R_n - \lambda E - H \tag{4}$$

where R_n is the net radiation, λE is the heat associated with the evaporation of water, and H is the sensible heat exchanged between the surface and the air. During the cold season, net radiation often dominates this relation [*Granger et al.*, 1977]; however, when the weather is overcast or foggy, the sensible heat exchange may be important. The evaporation term λE generally gives a net soil heat loss, but important exceptions do occur when the relative humidity is high; i.e., the presence of warm, moist air may result in the gain of heat by frozen soil or snow. Because the surface temperature of snow or ice does not rise above freezing, air dew-point temperatures above 0°C lead to condensation on snow or frozen surfaces with large releases of latent heat. A warm rain falling on snow causes rapid melt, not because of the energy carried by the raindrops themselves but because the rain favors a dew-point temperature above freezing. Water vapor condensing on the snow releases enough heat to melt 7 times the vapor's own weight of ice. Heat gained by vapor condensation is of the same order of magnitude as sensible heat gained from the air when the dew-point temperature is near the air temperature and above freezing (see, for example, the last two terms in (5)).

Soil Heat Flux From the Energy Balance

The average daily soil heat flux may be estimated from (4) by using daily air temperature extremes, solar radiation, snowfall, and several assumptions that deserve critical consideration.

Equation (4) may be written in more detail [*Campbell*, 1977, p. 61] as

$$G = R_n - \frac{\lambda \, \Delta\rho}{r} - \frac{\rho_a C_p \, \Delta T}{r} \tag{5}$$

where the terms correspond to those in (4) and definitions and dimensions of the variables are as follows:

- G soil heat flux (downward flow positive), W m^{-2};
- R_n net radiation, W m^{-2};
- λ latent heat of vaporization, J g^{-1};
- $\Delta\rho$ difference in water vapor concentration between the air at the soil surface and the air 2 m above the surface, g m^{-3};
- ρ_a density of air, g m^{-3};
- C_p specific heat of air, J g^{-1} °C^{-1};
- ΔT difference between the soil surface temperature and the temperature of the air 2 m above the surface, °C;
- r transfer coefficient, s m^{-1}.

Equation (5) is an instantaneous relationship; i.e., the true average values of λE and H for a time period of a few hours can only be approximately calculated from time average values of ΔT, $\Delta\rho$, and r because these variables change with time but not necessarily in phase with each other. Nevertheless, it was assumed that (5) holds, using daily mean values of all variables.

Net radiation may be expressed as

$$R_n = (1 - \alpha)S_t' + \sigma(T_a'^4 \epsilon_{ac} - T_s'^4 \epsilon_s) \tag{6}$$

where α is the shortwave reflectivity, σ is the Stephan-Boltzmann constant, S_t' is total shortwave radiation, T_a' and T_s' are the air and soil surface temperatures in degrees Kelvin, and ϵ is emissivity, ϵ_{ac} being the total incoming long-wave radiation and ϵ_s the outgoing long-wave radiation from the soil.

Using the approximation

$$T_s'^4 = (T_a' + \Delta T)^4 \simeq T_a'^4 + 4T_a'^3 \, \Delta T \tag{7}$$

and Penman's transform [*Campbell*, 1977, p. 120], (5) becomes

$$G_n = R_n' - \Delta T \left(4\sigma\epsilon_s T_a'^3 + \frac{C_p \rho_a}{r} + \frac{\lambda s}{r} \right) - \frac{\lambda}{r} (\rho_a' - \rho_a) - I_1 \tag{8}$$

where all quantities are now taken as daily averages. In (8),

- R_n' defined for convenience in programing as equal to $R_n + 4\sigma\epsilon_s T_a'^3 \, \Delta T$;
- s slope of the saturated vapor density curve;
- ρ_a' saturated water vapor concentration at temperature T_a;
- I_1 daily average heat flux that will be required to supply latent heat eventually to melt daily snowfall.

When snow is present, G_n in (8) is the heat flux across the snow-atmosphere interface.

The heat and vapor transfer coefficient for a nearly smooth surface is $r = 700/\bar{u}$, where $\bar{u}$ is the average wind speed in meters per second [*Campbell*, 1977, p. 138]. In this study, $r = 300$ s m^{-1} was chosen on the basis of average wind speeds, and $\epsilon_s = 0.98$ on the basis of a typically moist soil surface.

Using the daily minimum temperature T_m as an estimate of the dew-point temperature,

$$\rho_a' - \rho_a = 0.012(T_a^2 - T_m^2) + 0.34(T_a - T_m) \qquad (9)$$

where T_a is the mean daily air temperature in degrees Celsius. Equations (9) and (14) are best-fit quadratic relations describing data given by *Campbell* [1977, p. 150]. Similarly, in (8), s can be represented as

$$s = 5.6 \times 10^{-4} T_a^2 + 0.02 T_a + 0.34 \qquad (10)$$

when temperatures are within a few degrees of zero.

Taking $\alpha = 0.1$, (6) may be rearranged for any land slope and aspect as

$$R_n' = 0.9[S_d + K(S_t - S_d)] + \sigma T_a'^4(\epsilon_{ac} - 0.98) \qquad (11)$$

where S_d is the diffuse shortwave radiation and S_t the total shortwave radiation over a level surface. Daily values of S_d may be calculated from the average of hourly values given by *Campbell*'s [1977] equation (5.11). A constant daily value of 25 W m^{-2} was used for S_d, which agreed with measured values of S_t on winter days with heavy overcast. The factor K corrects for the effects of slope and aspect on the interception of direct shortwave radiation. Daily values of K at various latitudes can be interpolated from tables published by *Buffo et al.* [1972] and fitted to equations of the type

$$K = h[\sin (J + b_0)]^{1/2} \qquad (12)$$

where J is the Julian date and b_0 is the phase factor, taken as 95 for south slopes at this location. Values of h for each of the study sites are given in Table 1.

Campbell gives an equation [*Campbell*, 1977, p. 58, equation (5.14)] for estimating ϵ_{ac}, which was modified as

$$\epsilon_{ac} = 0.58\rho_a^{0.143} + \left(A - \frac{S_t}{S_{t0}}\right)(0.97 - 0.58\rho_a^{0.143}) \qquad (13)$$

where

$$\rho_a = 0.012 T_m^2 + 0.34 T_m + 4.82 \qquad (14)$$

in the neighborhood of 0°C, when T_m is a valid estimate of the dew-point temperature. Values for the potential shortwave radiation S_{t0} are given by *List* [1951] and expressed by the relation

$$S_{t0} = a_1 \sin (J + b_1) + c_1 \qquad (15)$$

TABLE 2. Weather Station Measurements During the Study Period

J	T_m, °C	T_a, °C	S_t, W m^{-2}	J	T_m, °C	T_a, °C	S_t, W m^{-2}
351	0.6	5.6	72	17	−8.3	−2.0	39
352	0	2.5	60	18	−6.7	0.3	39
353	−3.9	1.1	76	19	−1.1	5.0	67
354	−4.4	−0.3	71	20	0	2.5	88
355	−5.0	−0.6	49	21	−3.3	1.7	30
356	−5.0	−0.3	67	22	−3.9	−2.5	16
357	−5.0	0	24	23	−3.3	−2.0	20
358	−5.6	−2.5	54	24	−5.6	−3.7	31
359	−5.0	−3.1	26	25	−5.6	−3.7	11
360	0	1.7	19	26	−8.9	−6.1	97
361	1.1	5.9	71	27	−6.1	−3.1	34
362	−0.6	3.1	76	28	−9.4	−6.4	97
363	−3.9	−0.3	55	29	−9.4	−5.3	19
364	−7.9	−4.0	16	30	−8.3	−6.1	92
365	−5.6	−4.5	19	31	−8.3	−3.1	40
1	−12.2	−7.6	80	32	−6.7	−3.7	24
2	−12.2	−8.1	20	33	−4.4	−3.3	45
3	−8.9	−6.1	15	34	−3.3	−1.7	94
4	−8.9	−7.8	44	35	−2.8	−1.1	35
5	−17.8	−12.3	80	36	−4.4	−2.2	66
6	−15.5	−11.4	51	37	−3.9	−2.8	39
7	−12.2	−8.3	76	38	−4.4	−2.5	114
8	−15.5	−10.0	45	39	−5.0	1.1	54
9	−15.6	−12.5	65	40	−2.2	2.3	115
10	−17.2	−11.1	59	41	−2.2	2.8	56
11	−15.6	−9.8	34	42	0	6.1	113
12	−16.1	−8.4	44	43	0	5.3	57
13	−15.6	−8.9	22	44	0.6	7.5	119
14	−12.2	−6.1	24	45	3.9	6.7	123
15	−12.2	−5.3	34	46	2.8	6.1	109
16	−12.2	−4.5	60	47	2.2	7.5	111

Here J is Julian day, T_m the minimum air temperature, T_a the mean air temperature, and S_t the shortwave solar radiation. The correlation between T_m and measured values of dew-point temperature during this time was $r^2 = 0.46$.

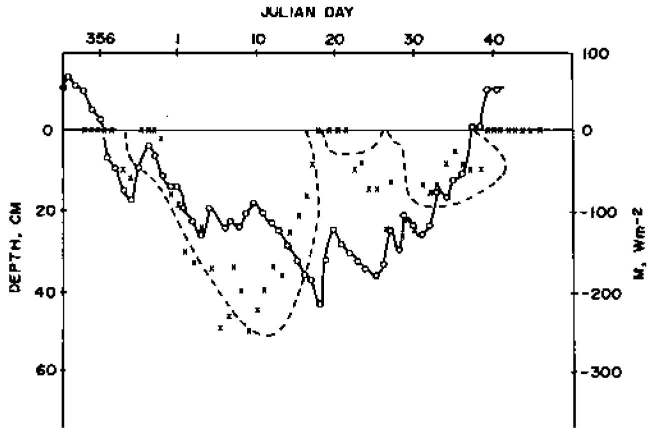

Fig. 1. Frost depth and sum of the soil heat flux deficits M for the bare south-facing slope. The dashed line shows the depth of frost penetration, the solid line shows M from (8), and the crosses give M from (18).

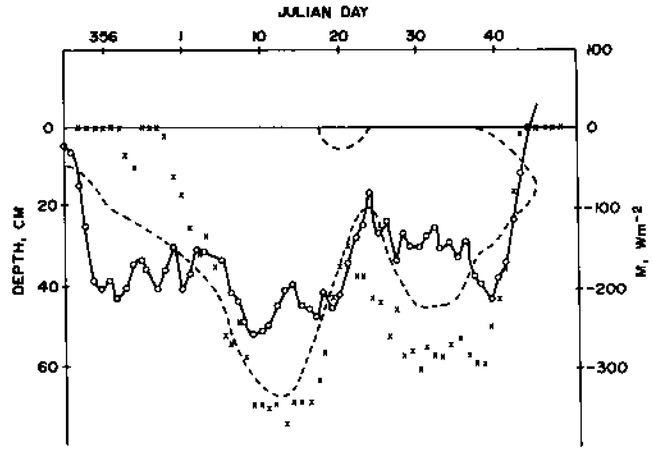

Fig. 3. Frost depth and values of M for the north-facing slope. Symbols are the same as those in Figure 1.

where $a_1 = 150$, $b_1 = 280$, and $c_1 = 230$ for our study location. The term $A - (S_t/S_{t0})$ corrects long-wave radiation for conditions different from a clear sky forming a complete hemisphere over the site. The constant A is a site factor with values near 1 that are sensitive to rough terrain and vegetative cover. Since A is chosen for each site to give the best fit between observed and predicted dates of soil freezing and thawing, it includes the effects of soil physical properties as well as a correction for all bias in the analysis resulting from assumptions and non-random measurement errors. Values of A for the five study sites are listed in Table 1.

One major difficulty with this energy balance approach is specifying accurate daily averages of ΔT. In this study, values of ΔT and R_n integrated over 2-hour time periods were measured on the bare south-facing site using a long-wave and a net radiometer. A linear correlation of these data led to the empirical relation

$$\Delta T = -0.03R_n' - 1 \qquad (16)$$

with a correlation coefficient $r^2 = 0.33$ between observed and measured values of ΔT. The r^2 value suggests that a constant value for ΔT, say, $-2°C$, might as well have been used. However, (16) was retained because it contains slope, aspect, and soil cover parameters that affect ΔT. When $T_a > 1°C$, $\Delta T = -T_a$, because a surface with frozen water is well buffered against temperatures above 0°C. This relation was included in the program with (16); i.e., the calculations were made on a hand-held programmable calculator.

Equations (9)–(16) and the associated assumptions allow one to estimate the average daily soil heat flux from (8) using the weather station data given in Table 2. The result can be used with (2) in (1) to predict whether or not the soil is frozen, provided there is no snow cover. Positive values of M indicate that the soil is thawed and the snow is melted. When M is negative, the amount of frozen water in the snow cover must be known before conclusions concerning the presence of soil ice can be reached.

Soil Heat Flux From the Surface Layer Approach

Equation (3) suggests a simple way to estimate daily soil heat flux from weather station measurements of daily mean temperature and snowfall. The average soil temperature T_0 a few centimeters beneath the surface will be proportional to the previous day's surface temperature. Combining this with the definition of ΔT gives $T_0 = (T_{a-1} - \Delta T_{-1})B^{-1}$, so that the temperature difference term in (3) becomes

$$T_s - T_0 = (T_a - \Delta T) - \left(\frac{T_{a-1} - \Delta T_{-1}}{B}\right) \qquad (17)$$

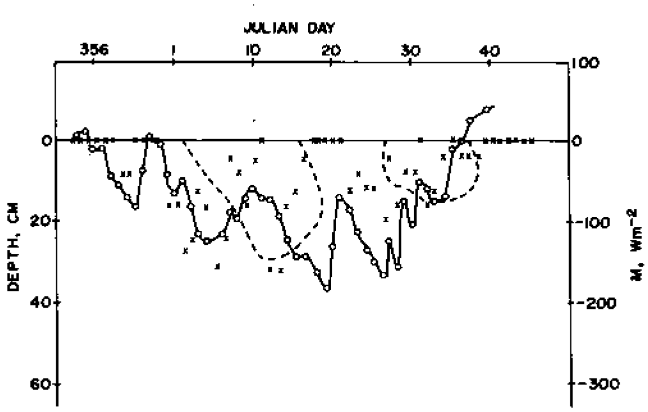

Fig. 2. Frost depth and values of M for the 18.5° south-facing slope. Symbols are the same as those in Figure 1.

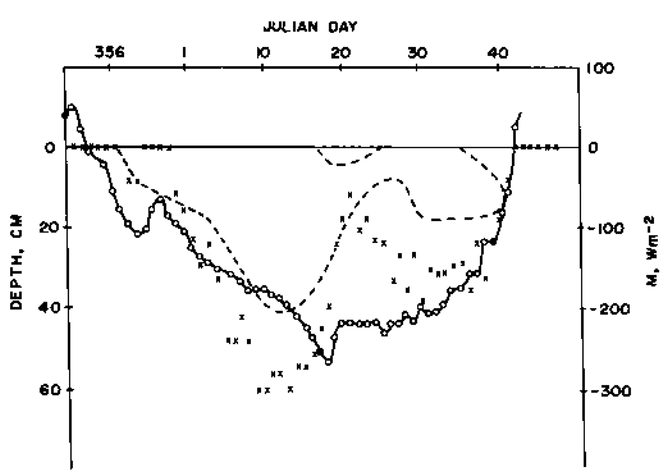

Fig. 4. Frost depth and values of M for the east-facing slope. Symbols are the same as those in Figure 1.

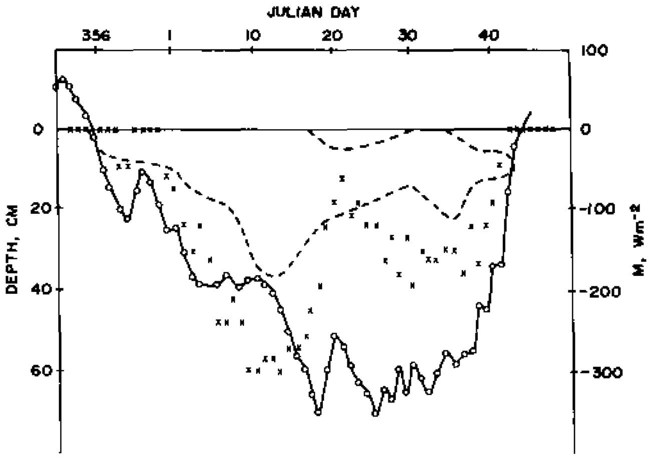

Fig. 5. Frost depth and values of M for the straw stubble plot. Symbols are the same as those in Figure 1.

where the -1 subscript indicates the previous day and B is the proportionality constant. Assuming that $\Delta T - \Delta T_{-1} \simeq 0$ and B is near 1, (3) becomes approximately

$$G_n = \frac{k}{l}\left(T_a - \frac{T_{a-1}}{B}\right)\left(1 - \frac{l_2}{l_2 + N}\right) \qquad (18)$$

where a damping term for snow cover $1 - [l_2/(l_2 + N)]$, has been included with the snow depth l_2. Since there was no significant snow cover during this study, the reader is referred to the work of *Anderson* [1976] for information on choosing a proper value for the constant N. The depth l was in the range 5–10 cm for these study sites. When $T_{a-1} > 0$, T_{a-1} is taken as zero in (18) because of the presence of ice in the soil. Likewise, when snow is present, the program must set the upper limit or T_a at zero. When (18) is used to evaluate (1), the freeze and thaw dates are determined by the value of B, while k/l determines only the amplitude of M. When $M > 0$, M was taken as zero in our program because the damping depth of unfrozen soil is much greater than it is when ice is present. Thus l should increase, while k generally decreases because of drainage away from the surface. The result is a decrease in G_n after the soil thaws.

RESULTS

The weather station data (Table 2) were used with the individual site constants (Table 1) to calculate values of M from (1) as a function of time. The results for each site are presented in Figures 1–5.

Values of M, using (8), did not predict the soil thaw periods between days 14 and 25 (Figures 1 and 2), nor did the curves produced by this algorithm match the depth of freezing curves very well after day 15 for any of the sites. Of course, the depth of freezing is not a unique function of the soil heat flux deficit because only part of the soil water freezes at 0°C. For example, in this soil, when the temperature falls from $-0.1°$ to $-1°$C, an additional 12% of the soil water freezes, and the heat released will correspond to a daily average flux of 40 W m^{-2} for each centimeter of water frozen. With an additional drop from $-1°$ to $-8°$C, another 5% of the soil water freezes [*Cary and Mayland,* 1972]. Obviously, the heat flux deficit M may vary by 50 W m^{-2} or more just because of soil temperature fluctuations near the surface without any change in frost penetration. The soil water content is also an important factor in the relation between freezing depth and values of M. The scales in Figure 1 were chosen so the freezing depth would approximately correspond to values of M. Owing to greater soil water contents, values of M in Figures 3–5 should be lower than the freezing curves but still approximate their shapes. The daily average heat flux measurements from transducers on the bare plot followed the freezing depth curve closely for the first 35 days but then did not predict the brief period of thawing that followed, possibly because of a faulty electrical circuit. A linear regression between the measured and the predicted heat flux from (8) had a correlation coefficient of $r^2 = 0.53$ during the first 35 days.

Values of M based on (18) predicted frozen soil better than those from (8), even though (18) does not include measurements of solar radiation. The correlation coefficient of G_n from (18) with measured soil heat flux on the bare site was $r^2 = 0.61$. As was pointed out by *Zuzel and Cox* [1975], air temperature is the best single parameter that one has for integrating the characteristics of microclimate. The most obvious error caused by the omission of S_t in the development of (18) is seen in Figure 3, where freezing of the north slope was not predicted until 10 days after it actually occurred. The values of M using G_n from (18) did not become negative at first because average daily air temperatures were above freezing; but the soil, which was in the shade of the north slope, was freezing from longwave radiation loss to clear skies. The effects of radiation could be included in (18) by starting with (17) and writing $\Delta T = f(S_t)$ and $\Delta T_{-1} = f(S_{t-1})$. This was not attempted here because ΔT was measured only on the bare, south-facing site. A better method for finding $\Delta T = g(S_t, T_a, T_m)$ (site properties) is needed for both (8) and (17). *Outcalt et al.* [1975] gave a numerical procedure for calculating average daily ΔT values at the snow-air interface. While their prediction of snowmelt was good, they did not report measurements of ΔT for comparison with calculated values. *Smith and Tvede* [1977] used the approach of Outcalt et al. to predict freeze-thaw dates and frost penetration under bare highways with no evaporation of water. They felt that this finite difference approach reproduced daily surface temperatures within 1° or 2° from daily air temperature, solar radiation, cloud cover, wind speed, air pressure, and some knowledge of the soil properties. Their predictions of freezing and thawing under several highways appear to have about the same accuracy as those in the study reported here. *Anderson* [1976] also presents a procedure for calculating ΔT values of a snow surface. His calculated values agreed within 1° or 2°C with those measured, agreement which is of about the same order of magnitude as our measured daily averages of ΔT values on the bare soil site. Other methods for finding ΔT of bare soil surfaces are available but require more detailed weather data than we use here [*Van Bavel and Hillel,* 1976].

APPLICATIONS

Since daily weather station records are available in many areas, (1) may be useful for analyzing data from past hydrologic events when the interpretation depends on knowledge of whether or not the soil was frozen. Equation (18) is obviously the preferred method for finding daily average values of G_n to be used in (1) when only limited weather station records are available. However, calculation of G_n from (8) is of more than passing interest because the difference in values of M from (8) and (18) gives the snowmelt. Based on the scatter of points in Figures 1–5, the uncertainty in such a calculation is in the neighborhood of 100 W m^{-2}, which corresponds to 2.5 cm of frozen water. Future improvements in the accuracy of (8) and

(18) could lead to a useful algorithm for predicting snowmelt as well as freezing and thawing of the soil.

Both (8) and (18) require a site constant which must be determined individually. Frozen soil data for this purpose can be obtained with frost tubes or thermocouples. If resources are available to make additional measurements other than T_a, T_m, and S_t, soil heat flux at representative sites will be a good choice for many purposes. Heat flux meters placed just beneath the soil surface and just below the zone of deepest frost penetration will give values for M from a voltage integrator.

The soil heat flux deficit M may be used to estimate the depth of frost penetration if one also knows the soil water release curve as well as the water content and soluble salt distribution with depth [*Cary and Mayland*, 1972]. In like manner, if one has this information on the soil properties and is measuring the frost penetration with frost tubes or thermocouples, M can be estimated. These values of M might then be used in the algorithms presented here to predict the type of weather that must occur before the soil will thaw, a key factor in flood and water storage forecasting.

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