that for a given water level in the submerged state, the head losses are proportional to the square of the discharge, in general

$$\frac{H_1}{H} = 1 - \left(1 - C_D^{2/3}\right)\left(\frac{Q}{Q_o}\right)^2 \quad \text{(69)}$$

Writing the energy equation upstream of the throat

$$H = y_1 + \frac{Q^2}{2g\, b_3^2\, y_1^2} \quad \text{(70)}$$

in which b_3 is the width of the entrance transition (2.5 ft) at the upstream tapping positions and substituting from Eq. 67, it follows that

$$\frac{H}{y_1} = 1 + \frac{4}{27}\left(\frac{b_2}{b_3}\right)^2\left(\frac{Q}{Q_o}\right)^2\left(\frac{H}{y_1}\right)^3 \quad \text{(71)}$$

Solving Eqs. 63, 65, 66, 69 and 71 for $b_2/b_1 = 1/3$ and a constant value of C_D of 0.951 corresponding to $h = 1.8$ ft, a theoretical reduction factor—submergence relationship has been determined. This is plotted on Fig. 24 and shows reasonable agreement with the experimental points.

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AUGUST R. ROBINSON,[21] F. ASCE.—This paper presents another contribution to the understanding of flow phenomena for measuring flumes. The particular regime of submerged flow has been largely neglected, although there have been a number of attempts to establish criteria and methods for analyzing submerged flow as referenced by the authors.[6,7,9,10,11] The common approach has been to develop the theory to a point and then proceed with an empirical and experimental approach. In this manner it has been possible to develop relationships which are usable for particular flumes of given geometry. However, because of the empirical approach, it is not possible to use the relationships for designs other than those for which they were developed. This paper also summarizes a study for particular design geometries and has contributed to the knowledge of submerged flow. There still exists the need for good, sound theoretical developments so that the relationships include the geometry of the measuring flume, as well as those items usually assumed to be of negligible effect, such as frictional losses and nonuniform velocity distribution. The geometry variables have been included for the free flow case by Ackers and Harrison[9] with apparent success. Flume geometry is important since such things as length of throat, contraction ratio (B), shape of section

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(rectangular or trapezoidal), and geometry of the downstream diverging section all contribute to submergence effects.

Some discussion of the needs for measuring flow under submerged conditions is in order. For the field situation, measurements to correct for the submerged condition are rarely made because of lack of knowledge or indifference. Failure to correct for submergence may result in errors in flow measurement that may exceed 30% in extreme cases. The errors always indicate more flow than is actually occurring. Measuring flumes should generally be set so the operation is in the free-flow range, but, for many reasons, this is not always possible. It has been estimated that many of the measuring flumes now in use operate submerged for at least a part of the flow season.

The authors used a specially constructed flatbottomed, Venturi-type flume for study. Although the throat width was not specified, it was apparently 1-ft for the structure shown in Figs. 6 and 7, and had a short converging section together with a long throat relative to the width. It is interesting to compare the free-flow discharge equation (Eq. 13), evidently obtained from the six points shown in Fig. 8, with those for a 1-ft Parshall flume and a standard rectangular weir. The exponent of the depth (1.525) is very near 1.5 and is practically identical to that for the 1-ft Parshall (1.522). The exponent of the depth in the discharge equation for Parshall measuring flumes is different from 1.5 since the discharge coefficient as well as the discharge is a function of the depth. There is a question as to the large difference in the coefficient (2.87) from that for the standard suppressed 1-ft rectangular weir (3.33) and the Parshall flume (4.00). With the indicated geometry, it would be reasonable to expect the value of the coefficient to be between that of the weir and the Parshall flume. In a more recent publication, the authors present information on a flume of similar geometry with the exception that the throat section has been eliminated (17). The points for measuring h_a and h_b were also changed slightly. In this case, for a 1-ft flume, the exponent of h_a is 1.56 and the coefficient is 3.50.

The authors have used dimensional analysis to develop parameters for relating the effects of submerged flow discharge. Graphical analysis has been used, to a large extent, in developing the relationships. The material as presented is difficult to follow because of the number of plots and related analyses. When using dimensionless parameters, care must be taken to assure that the apparent correlations are not spurious (14). One relationship which might be questioned is given in Fig. 9. Essentially the parameters of the abscissa and ordinate contain the same variables. i.e., y_1 and y_3 since y_m is a function of y_1 and y_3 also. In this case, the flume discharge is also a function of y_m as given in Eq. 21. It would be reasonable to expect that the magnitude of the discharge is important and results in the obvious nonlinear relationship shown in Fig. 9. Eq. 18 which is obtained by assuming a linear relationship is questionable.

Plots such as those shown in Figs. 5 and 13 are a unique development and represent a commendable contribution by the authors. One reason why submergence corrections are not made is the complicated procedures required by existing methods. Any procedure that simplifies the computations for field personnel is welcomed and will increase the frequency of use. The so-called "three dimensional" plots of the authors are a step in this direction. It should be pointed out that the "free flow region" shown in Fig. 5 is misleading, since free flow applies only to the curve (line) representing the free flow equation and the limit of submerged flow. Another useful feature of plots such as Fig. 13 is in determining the approximate energy loss through the structure for a given flow and degree of submergence. If the locations for y_1 and, particularly y_3, are far enough upstream and downstream so that the difference $(y_1 - y_3)$ represents the approximate change in total energy, then this is an estimate of the head loss through the structure. The flow geometry at y_1 and y_3 must be approximately equal.

The relationship for determining the effect of submergence as given in Fig. 5 can be further simplified and is shown in Fig. 25. This plot was taken directly from the information given in Fig. 5 for the 2-ft Parshall flume. The discharge Q_0 is the observed discharge (obtained from calibration tables) for

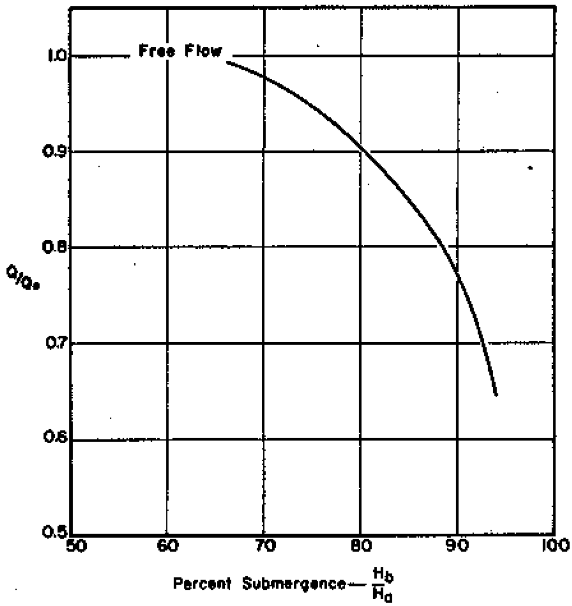

FIG. 25.—EFFECT OF SUBMERGENCE ON DISCHARGE FOR 2-FT PARSHALL FLUME

a depth that has been increased because of submergence. The ratio Q/Q_0 is actually a correction factor which can be applied to the observed discharge in order to obtain the correct flow. The procedure for using this plot and information on other sizes of Parshall flumes is discussed in an earlier publication.[11] Similar procedures for trapezoidal measuring flumes have been developed (15,16).

The authors have stated that submergence is defined as the ratio of a downstream depth to an upstream depth. They state that "in actuality, the ratio of any flow depth measured downstream from y_m (the minimum depth) to any depth upstream from y_m can be used as the submergence S." It must be clearly understood that these points must be selected and calibrations using depths at these locations are necessary for correct usage. The locations for depth determinations must be clearly defined for each particular flume.

The authors have equated their Eqs. 13 and 22 in order to obtain a value of submergence at which the transition from free flow to submerged flow occurs. Eq. 23 was obtained and solved by trial and error for the transition submergence. The value of 0.893 is much higher than would be expected since it has generally been observed that submergence effects usually begin at values ranging from 0.70 to 0.80 for flumes of this size. Engel[5] showed that the point of critical submergence may vary from 0.67 to 0.90 depending on the geometry of the throat and diverging sections. Critical submergence for the 2-ft Parshall flume is shown to be 0.66 in Fig. 5. A study of Fig. 25 indicates that the effect is gradual for a particular flume and some degree of submergence may be allowed before corrections are necessary. There is also evidence that submergence effects are, to some extent, dependent on the magnitude of discharge.

The purpose and use of Fig. 14 are questioned (14). In an attempt to relate the denominators of the two submerged discharge equations (Eqs. 10 and 22), the functional relationships of $\phi_m(S)$ and $f(S)$ given in Eqs. 26 and 27 were used. The authors stated that the value for C_2 in Eq. 27 equals zero when the submergence ratio increases to one and proceed to graph values of $\phi_m(S)$ and $f(S)_{C_2=0}$ for the case of C_2 equaling zero. It should be noted that the values of both $\phi_m(S)$ and $f(S)$ approach infinity for this condition and finite values of S should not be assigned. A question is also asked as to the meaning of constriction ratios (B) of 0.0 and 1.0 in Fig. 14. If B is zero, then b_2 is zero and there is no throat width. For B of unity, b_2 equals b_1 and there is no constriction.

In summary, the authors have made some progress in the solution of the submerged flow case for measuring flumes. However, the method of analysis is not clear and is incomplete and leaves the reader with a number of questions regarding the analysis. In an attempt to relate a theoretical approach with an empirical study, certain relationships were developed which appear to be questionable.

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