ABSTRACT: The primary source of error in properly calibrated, constructed, and installed flow measurement devices is due to reading error or uncertainty. Head reading uncertainty in small V-notch flumes and submerged orifices is measured in the field as ±3mm with no consistent variation with reading. Elapsed time measurement uncertainty for volumetric measurements increases with the square root of the time. The sensitivity of flow measurement uncertainty to head or time reading uncertainty is proportional to the ratio of the device discharge equation exponent to the reading. Furrow flow measurement uncertainty varies with the device and flow rate, but generally exceeds ±5% and often exceeds ±10%. Maintaining uncertainty below ±10% requires flume measurements in the upper 50% of their range, orifice measurements with head readings greater than 13mm, and volumetric measurement elapsed times greater than 4 seconds.

FLOW MEASUREMENT UNCERTAINTY

Any measurement process involves a degree of uncertainty, sometimes referred to as inaccuracy or potential error. If many measurements are made of a value, they will randomly scatter around the true value if all systematic errors are eliminated or corrected. The standard deviation of the measurements \( s \) is a measure of the scatter and is used to determine the uncertainty of the measurement \( E \), or the interval within which the true value lies with a certain confidence level or probability. The uncertainty of a measurement can be calculated by:

\[
E = \frac{ts}{\sqrt{n}} \quad (1)
\]

where \( t \) = the Student's \( t \)-statistic (two-tailed), dependent upon the degrees-of-freedom associated with estimating \( s \) (usually \( n-1 \)) and the desired probability level; and \( n \) = the number of measurements used to calculate the mean. The \( t \)-statistic can be found in basic statistics texts.

Often, the uncertainty of a measurement or device is given in terms of a value implied to be the maximum error. *International Standard #5168 (ISO 1978)* defines this “maximum” deviation in flow measurement as the 95% confidence interval.

Bos et al. (1984), Bos (1976), and ISO (1978) discuss flow measurement error sources and their effects. The major sources of uncertainty in head:

1Agric. Engr., USDA-Agric. Res. Service, Route 1, Box 186, Kimberly, ID 83341.

2Statistician, USDA-ARS, Western Regional Res. Ctr., 800 Buchanan St., Albany, CA 94710.

Note. Discussion open until October 1, 1988. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on December 4, 1986. This paper is part of the *Journal of Irrigation and Drainage Engineering*, Vol. 114, No. 2, May, 1988. ©ASCE; ISSN 0733-9437/88/0002-0244/$1.00 + $.15 per page. Paper No. 22412.
discharge-type open channel flow measurement devices (flumes, weirs, and orifices) are:

1. Uncertainty in the device calibration, or equivalently, uncertainty in the dimensional duplication of the calibrated device, usually limited by calibration standards and construction tolerances.
2. Uncertainty in the duplication of calibration flow conditions, generally limited by standard installation procedures.
3. Uncertainty in the reference elevation of the gage.
4. Uncertainty of the head reading due to gage resolution, water surface waves, and light reflection or meniscus interference.

Source 1 uncertainty will cause a systematic error in a given device at a given flow rate. Sources 2 and 3 can cause systematic errors for a given device setting. Procedure-related source 3 and 4 uncertainties (i.e., using an unlevel surface to level a flume) can also cause systematic errors. Systematic errors must be added to random uncertainties to estimate the total measurement uncertainty. However, if several devices of the same type are used interchangeably, source 1 uncertainty due to dimensional variations becomes a random error source. If the device is reset or the critical factors (leveling, sediment accumulation, etc.) are monitored and corrected for before each reading, source 2 and 3 uncertainties will also tend to be random. If properly calibrated and dimensioned devices are used according to standard procedures, the remaining uncertainty will be primarily the result of sources 3 and 4 and will be random.

In addition to head-discharge-type devices, flow measurements can also be made volumetrically by recording the time required to fill a container of known (calibrated) volume or to add a mass of fluid to a container. For volumetric measurements, the error sources are container calibration (systematic), which should be small, and timing. Timing uncertainties result from imprecise timer starting and stopping and the judgement decision of when the container is full. These random timing errors are considered equivalent to the flow depth or head measurement errors in the other devices.

The relationship between the variance (the square of the standard deviation) of a device head reading and the variance of a flow measurement can be approximated by the first term of a Taylor series expansion (Kendall and Stuart 1963):

\[ V_Q = \left( \frac{dQ}{dR} \right)^2 V_R \]  

where \( V_Q \) = variance of the flow measurement; \( V_R \) = variance of the reading; \( Q \) = flow rate, and \( R \) = reading. Inserting Eq. 2 into Eq. 1:

\[ E_Q = \frac{ts_Q}{\sqrt{n}} = \frac{1}{\sqrt{n}} \left( \frac{dQ}{dR} \right)^2 V_R \]  

where \( E_Q \) = uncertainty of the flow measurement; \( s_Q \) = standard deviation of the flow measurement = \( \sqrt{V_Q} \); \( s_R \) = standard deviation of the device reading = \( \sqrt{V_R} \); and \( E_R \) = uncertainty of the reading.

Consequently, the uncertainty in a flow measurement is proportional to the uncertainty in the device reading. The derivative term, \( dQ/dR \), is also called the sensitivity of the device (Bos 1976; ISO 1978).

The objective of this study is to determine the flow measurement uncertainties under field conditions of three types of furrow flow measurement devices and to determine methods to minimize uncertainty.

**PROCEDURE**

During three trials conducted on three different days, nine technicians each measured the flow at the head of the same 10 furrows with a 3.58-L (1-gal) bucket, three-holed acrylic orifice plates (Trout 1986a, 1986b) and fiberglass long-throated 60° V-notch flumes (Robinson and Chamberlain 1966; ASAE 1983; Trout 1986c). Measurement device calibrations are given in Table 1. Flow rates into each furrow were preset and held constant during each trial at between 4–60 L/min. All technicians were trained and had some experience in the use of the devices.

The volumetric measurement was done in pairs with one member holding the bucket and the second operating a stopwatch. Time-to-fill readings were recorded to the nearest 0.1 sec. In trials 1 and 3, each individual measured each furrow four times.

The orifice plates were preinstalled and the appropriate orifices selected to produce a submerged flow head loss of between 10–40 mm. Readings of the upstream and downstream water surface elevation difference were made with portable differential point gages (Trout 1986b) to the nearest millimeter. Gage levelness was only visually assessed.

In trials 1 and 3, the V-notch flumes were installed, leveled, and read by each individual. Each individual randomly selected two or three flumes to use from a group of 12. In trials 2 and 3, flumes were preinstalled in the furrows and each individual checked and altered, if necessary, its installation (leveling, sediment accumulation, water leakage) before taking the reading. Flume flow depth readings were taken from the flume side gage, which has markings at every 2 mm but was estimated to the nearest millimeter, and with a vertical point gage (Trout 1986c) with a 1-mm resolution. Both depth readings were made in the flowing water at the flume gaging point. Side-gage readings were reduced by 1.5 mm, as indicated in Table 1, to correct for a systematic bias caused by the water surface meniscus. The fiberglass flumes are mass produced on precision metal forms.

![Table 1. Furrow Flow Measurement Device Calibrations](image_url)

<table>
<thead>
<tr>
<th>Type</th>
<th>Device Description</th>
<th>Reading Resolution (mm)</th>
<th>Discharge Equation (m^3/s)</th>
<th>Sensitivity (dQ/dR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volumetric</td>
<td>3.78 L-bucket and stop watch</td>
<td>0.1 sec</td>
<td>( Q = 227R^{0.73} )</td>
<td>227R^{0.73}</td>
</tr>
<tr>
<td>Flume</td>
<td>Fiberglass 60° V-notch</td>
<td>1 mm</td>
<td>( Q = 0.00543(R-1.57)^{0.83} )</td>
<td>0.00543R^{0.83}</td>
</tr>
<tr>
<td>Orifice</td>
<td>Three-hole acrylic orifice plate</td>
<td>1 mm</td>
<td>( Q = 0.00411(R)^{0.85} )</td>
<td>0.00411R^{0.85}</td>
</tr>
</tbody>
</table>

*With sloping side gage (line 1) and vertical point gage (line 2).

\( d = \) orifice diameter (mm); \( C_d = 0.625. \)

Square root transformation.
Table 2 lists the individual trial reading-variance component, the variance best estimate, the reading standard deviation for each measurement device and the V-notch furrow procedure. In each trial, the square root of the V-notch furrow variance reading was about equal to the mean reading. The standard deviations were calculated using the best estimate for each measurement device. The reading standard deviations were plotted against the average reading for each furrow to determine if the variance varied with the mean.

Table 2. Furrow Flow Measurement Device Reading Variance Component Estimates

<table>
<thead>
<tr>
<th>Device</th>
<th>Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V-notch</td>
<td>0.180</td>
<td>0.190</td>
<td>0.150</td>
<td>0.140</td>
<td>0.130</td>
<td>0.140</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>Furrow</td>
<td>0.180</td>
<td>0.190</td>
<td>0.150</td>
<td>0.140</td>
<td>0.130</td>
<td>0.140</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>Subject</td>
<td>0.180</td>
<td>0.190</td>
<td>0.150</td>
<td>0.140</td>
<td>0.130</td>
<td>0.140</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>Reader</td>
<td>0.180</td>
<td>0.190</td>
<td>0.150</td>
<td>0.140</td>
<td>0.130</td>
<td>0.140</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>Paper</td>
<td>0.180</td>
<td>0.190</td>
<td>0.150</td>
<td>0.140</td>
<td>0.130</td>
<td>0.140</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Note: The variance component estimates are shown for both the V-notch and furrow devices. The subject variance component is the largest component in the variance model. The reader variance component is the second largest component. The paper variance component is the third largest component. The subject variance component is the largest component in the variance model. The reader variance component is the second largest component. The paper variance component is the third largest component.
FIG. 1. Deviations of Mean Measured Flow Rates from Standard Measurement in Trial 3

even. The reduction in random uncertainty will be roughly equivalent to the increase in systematic error. For truly independent measurements, the device and installation must be changed.

Table 2 indicates that when the head reading was made on the same flume setting with both the 60°-sloped side gage and the portable point gage, the reading standard deviations were similar. The side-gage readings were expected to be more difficult to judge due to the curved water surface meniscus against the gage. Apparently the portable point gage is sufficiently difficult to use that the advantages of its precision are not always realized in the field.

For both gages, the reading variability \( s_R \) tended to increase slightly with the reading, although the relationship was not consistent so no variation of \( s_R \) with \( R \) was assumed. The Froude number at the gaging point varied only from about 0.1 at 4 L/min to about 0.2 at 60 L/min, so the water surface was fairly smooth over the measured range.

Although the total reading variance of one flume test was about four, the remainder were below two, so the reading standard deviation of the V-notch flumes, including effects of flume construction, installation, and gage reading, will be estimated as 1.5 mm.

Orifice-plate differential point-gage head-reading variances were consistent among the trials, and the best estimate standard deviation was 1.3 mm. The reading standard deviations did not vary consistently with measured differential head or orifice size.

Although all orifice plates were preinstalled, Fig. 1 and Table 3 show that little systematic error resulted from orifice-construction or plate-installation differences. None of the means deviated from the standard measurement by more than the 95% confidence limit, although the small positive average deviation (0.6%) indicates a possible slight positive error in the orifice discharge coefficient.

The reading standard deviation for the volumetric method increased at a decreasing rate from about twice the resolution (0.2 sec) at low readings (high flow rates) to about seven times the resolution at high readings. The increase is likely due to the increasing time required to raise the water level in the bucket a depth increment. Consequently, any difference in judgement of when the bucket is full causes greater reading variability at larger time readings. The decreasing rate of increase could result from the reduced judgement differences at lower flow rates (higher readings) due to less water surface turbulence. A square root transformation of the readings was used to stabilize the variances over the reading range (Box et al. 1978), and the variances listed in Table 2 are of the transformed data. The volumetric method sensitivity, \( dq/dR \), must also be adjusted for the transformed readings.

\[
\frac{dQ}{d\sqrt{R}} = \frac{d(227\sqrt{R}^{-2})}{d\sqrt{R}} = 454\sqrt{R}^{-3} = 454R^{-1.5} \quad (4)
\]

An estimate of the reading standard deviation \( s_R \) (untransformed basis) from \( s_{\sqrt{R}} \) would be

\[
s_R = 2\sqrt{R} \cdot s_{\sqrt{R}} \quad (5)
\]

Thus, \( s_R \) is estimated to vary from 0.19 sec at \( R = 3.8 \text{ sec} (Q = 60 \text{ L/min}) \) to 0.72 sec at \( R = 57 \text{ sec} (Q = 4 \text{ L/min}) \).

Fig. 1 and Table 3 show a small positive bias in the volumetric measurements, possibly resulting from underestimation of bucket fullness. Adjustment for this apparent systematic error brings all mean deviations to within the 95% confidence limit of the standard flow.

When multiple readings are made, the precision of the estimated flow rate is increased. The standard deviation of the mean, \( s_x \), can be estimated by

\[
s_x = \sqrt{\frac{rV_{50}V_R + rV_{FS} + V_e}{br}} \quad (6)
\]

*Root-mean-squared deviation = [\((\text{mean measured flow}-\text{standard flow})^2\)]^{1/2}.*

TABLE 3. Average Deviations of 10 Mean Measured Flow Rates from Standard Flow Rate in Trial 3

<table>
<thead>
<tr>
<th>Device</th>
<th>Average absolute deviation</th>
<th>Root-mean-squared deviation*</th>
<th>Maximum deviation</th>
<th>Average deviation</th>
<th>Number of deviations greater than 95% confidence limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Flume-preset side gage</td>
<td>3.8</td>
<td>4.7</td>
<td>+11</td>
<td>+3.8</td>
<td>4</td>
</tr>
<tr>
<td>Side gage point gage</td>
<td>2.2</td>
<td>2.3</td>
<td>-5</td>
<td>+1.0</td>
<td>1</td>
</tr>
<tr>
<td>Flume-individual set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Side gage</td>
<td>2.1</td>
<td>2.8</td>
<td>-6</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>3.78-L container</td>
<td>1.1</td>
<td>1.4</td>
<td>+2</td>
<td>+0.9</td>
<td>2</td>
</tr>
<tr>
<td>Orifice plate</td>
<td>1.6</td>
<td>2.0</td>
<td>+4</td>
<td>+0.6</td>
<td>0</td>
</tr>
</tbody>
</table>

249

250
where \( V_s \) = subject variance component estimate; \( V_R \) = reading variance component estimate; \( V_{FS} \) = furrow x subject variance component estimate; \( V_c \) = furrow x reading subject variance component estimate; \( r \) = number of readings per subject; and \( b \) = number of subjects (readers). For tests 1 and 3, utilizing the mean of four readings for an individual would reduce the standard deviation of the transformed readings from 0.046 to 0.035 or by 24%.

For all devices and flow rates, the device sensitivity times the reading standard deviation predicted the flow rate measurement standard deviation to within ±8%, and in most cases to within ±4%, so the Taylor series approximation used in Eq. 2 appears sufficiently accurate.

**ANALYSIS**

Flow measurement uncertainty in properly calibrated and used devices is primarily dependent on the uncertainty in the reading and the sensitivity of the device. Thus, to minimize uncertainty, readings must minimize uncertainty relative to the reading, and devices should be chosen and sized to minimize sensitivity in the measured flow range.

Reading uncertainty will be minimized in head:discharge-type devices by: (1) Selecting and designing the device such that Froude numbers at the gaging point will be low and/or taking readings in still water such as in a stilling well; and (2) carefully leveling the device or otherwise establishing the reading datum or translocating the gaging point, via a pressure line, to a stilling well near the device control point (Bos et al. 1984). Both stilling wells and gaging point translocation increase the bulk and cost of measurement devices. A mechanical device such as a point-gage can increase the precision of a reading, especially if used in a stilling well. However, the reading variability measurements indicate that, for the small-portable point gages used on the flowing water surface, the potential improvement was not realized. From the field measurements made, the source of uncertainty cannot be separated between that due to errors in gaging reading and that due to errors in gage datum elevation.

The head loss readings for an orifice plate can be measured in relatively still water near the plate due to the large amount of flow contraction through the orifice. Thus, reading uncertainty should be low due, both to the tranquil water surface and to decreased sensitivity to gage leveling. A gage datum one degree out of level relative to the device control point will cause only a 0.7-mm error in a submerged orifice plate reading when the gaging points are 40 mm apart, but a 3-mm error in a furrow flume reading when the gaging point is 200 mm from the flume control point. The more consistent and smaller average measured reading uncertainties of the orifice plate differential head measurements compared to the V-notch flume head measurements reflect these potential advantages.


where \( a \) and \( u \) = the device discharge coefficient and exponent, respectively. The exponent values are about 2.5 for sloping-sided flumes and weirs, 1.5 for rectangular flumes and weirs, 0.5 for orifices, and −1 for volumetric measurements. The coefficient is proportional to the control section cross-sectional area of a device. By differentiating Eq. 7, the sensitivity is given

\[
dQ \, dR = u a R^{u-1} \tag{8}
\]

The sensitivities of the tested devices are listed in Table 1. By dividing Eq. 8 by Eq. 7 and inserting into Eq. 3, an expression for the relative flow measurement uncertainty \( E_{Qp} \) is derived

\[
E_{Qp} = \frac{E_Q}{Q} = \frac{E_R}{R} \tag{9}
\]

Thus, the relative uncertainty in the measurement of a flow \( Q \) is proportional to both \( u \) and the ratio \( E_R/R \).

**FIG. 2** shows the measured flow measurement uncertainty of the three studied devices with \( s_R = 1.5 \) mm for the flumes, 1.3 mm for the orifices, and 0.046 \( \cdot \) 2V/R for the volumetric readings. Since the derived reading standard deviations were determined with a large number of measurements, \( r = 2.0 \) for the 95% confidence limit, so \( E_R \) for a single measurement \( n = 1 \) was calculated as \( 2s_R \). Note that the figure projects uncertainties above the 60-L/min measured range on the assumption that the measured \( s_R \) values are valid up to 100 L/min. **FIG. 2** shows that projected furrow flow measurement uncertainties vary widely with measurement device and head loss.

Due to the low sensitivity of orifice plates, the head loss through an orifice increases rapidly with flow rate. The head loss range must be limited to prevent excessive backwater conditions upstream due to high head loss and low \( E_R/R \) ratios and thus high uncertainty at low head loss (Trout 1983,
1986b). Consequently, when the head loss in the example shown in Fig. 2 exceeds 45 mm, orifice area is increased 2.1 times, which results in a minimum head loss of 10 mm and thus a maximum relative uncertainty of 13%. Trout (1986b) describes the design process for such multihole orifice plates. As Fig. 2 shows, three orifice sizes are required to measure from 10–100 L/min within these head loss constraints. Note that the lower head loss limit establishes the maximum uncertainty limit. If the lower head loss limit were 15 mm, the maximum uncertainty would be 8.5%.

The 3.78-L volumetric measurement is most accurate in the lower flow range but least accurate in the upper range, due to rapid container filling and small time readings. To maintain accuracy at higher flows, a larger container must be used. Sensitivity is inversely related to container size. For example, the uncertainty of the standard flow rate measured with the 15.7-L container would be about half (V/3.78/15.7) the uncertainty shown in Fig. 2, assuming the average remains the same. Container shape (surface area-to-volume ratio) affects the depth increase rate and thus $S_{R}$.

The V-notch flume is the least accurate device in the lower flow range, primarily due to the high discharge equation exponent value $u$. Its 95% uncertainty limit is greater than 10% up to 50% of its approximately 100-L/min capacity. Flume sensitivity could be decreased (and thus accuracy increased at a constant $S_{R}$) by increasing the head or flow depth by decreasing the throat width or by using a throat with side rather than bottom contractions, and by decreasing the discharge equation exponent $u$ by using a rectangular rather than trapezoidal control section. However, greater flow depths generally require greater head loss for critical flow conditions and thus increase upstream flow depths, and less sensitivity decreases the measurement range.

The furrow flow measurement uncertainties shown in Fig. 2 do not include any systematic error. They apply to properly calibrated and used devices. Device calibrations are often reported to be accurate to within ±2%. The measurements indicate the tested devices met this criterion. Systematic error can be estimated by comparing measurements made with different devices.

**SUMMARY**

Furrow flow measurement device design or selection will require evaluating the trade-off between required accuracy, desired range, and allowed head loss. Volumetric measurements are potentially accurate but require several centimeters of free-falling water. The required fall height increases with container size and thus range and accuracy. Submerged orifice measurements are potentially accurate due to the low sensitivity, but range is consequently narrow and head measurement is equal to the head loss. Flumes measure a wide range of flows and, with long-throated flumes, required head loss is often less than 20% of the head measurement. However, flume flow measurement is most sensitive to head reading errors, and sensitivity increases with range and decreasing allowable head loss.

Flow measurement accuracy is also dependent upon the accuracy of the head or time measurement. Measured head reading standard deviations with side or point gages on the flowing water surface were in the range of 0.2 mm and thus 95% uncertainty limits were 2–4 mm. Reading uncertainty in head:discharge devices can be reduced with stilling wells, especially if the head is translocated to near the device control point. These improvements increase device bulk, complication, and cost. Head reading uncertainty will also depend upon the training and care of the reader. The readers in this study were trained and closely supervised.

With the measured reading uncertainties, flow measurement uncertainties of the tested devices often exceeded 10%. This is larger than is commonly claimed and strongly affects the uncertainties of infiltration and irrigation evaluation measurements. A companion paper (Trout and Mackey 1988) analyzes the effects of infiltration measurement.

**CONCLUSIONS**

1. Head measurement uncertainty in small head:discharge-type flow measurement devices measured in the field by closely supervised technicians on a relatively smooth flowing water surface was ±3 mm.
2. Volumetric time measurement uncertainty increased with the square root of the container fill time.
3. Flow measurement uncertainty with three tested devices generally exceeded 5% and exceeded 10% over parts of their range.
4. Desired measurement range or allowed head loss constraints must often be limited to achieve desired flow measurement accuracy.

**APPENDIX I. REFERENCES**


**Appendix II. Notation**

The following symbols are used in this paper:

- $a$: coefficient of measuring device discharge equation;
- $b$: number of subjects (readers);
- $E$: measurement uncertainty or one-half confidence interval;
- $E_{QP}$: relative flow measurement uncertainty;
- $n$: number of measurements used to calculate mean;
- $Q$: flow rate;
- $R$: device reading such as flow depth in flume;
- $r$: number of readings per subject;
- $s$: standard deviation $= \sqrt{V}$;
- $t$: Student's $t$-statistic (two-tailed);
- $u$: exponent of measuring device discharge equation;
- $V$: variance;
- $V_e$: furrow x reading x subject variance component estimate;
- $V_{FS}$: furrow x subject variance component estimate;
- $V_R$: reading variance component estimate; and
- $V_S$: subject variance component estimate.

**Subscripts**

- $R$: reading;
- $Q$: flow; and
- $\bar{x}$: mean.