PLASTIC theory, which predicts a constant ratio between stress and strain, is used as a guideline for the engineering use and testing of soils. The mechanical analog for an elastic medium is simply a spring or the electric circuit equivalent is a capacitor. Soil can be considered as an elastic medium only under conditions of very small strain or high compaction. A technique used to describe slightly more complex materials is to join a spring and a dashpot or, electrically, a capacitor and a resistor to form a viscoelastic model, one with both viscous and elastic components. One advantage of the viscoelastic model over an elastic one is that the viscous term is included to account for energy loss that occurs within the material as it is deformed.

The plastic model of a spring and a sliding friction block connected in series also permits energy loss. This model is used extensively in soil mechanics, however, it is handicapped by the absence of a mechanism permitting time-dependent, stress-strain relationships. Since many properties of agricultural soils are time-dependent, the application of a viscoelastic model to such soils is discussed in this paper.

REVIEW OF PROCEDURE

For simple material, combining a pair of elements—one spring and one dashpot—may satisfactorily describe its behavior. However, for most materials such as plastics, asphalts, or soils, models composed of a large number of spring and dashpot pairs are necessary to accurately approximate observed stress-strain behavior.

Mathematically, it is convenient to enlarge the number of pairs in these models to an infinite number. Such models are known as generalized Maxwell or Voigt models, depending on whether the spring and dashpot of each pair are connected in series or parallel.

Between these two generalized models, any conceivable combination of viscous and elastic behavior may be matched. The use of this general viscoelastic theory for describing uncompacted soils has been proposed by Waldron (1964, a and b) Kondner and Ho (1965) and Krizek (1968). Laboratory tests by Waldron (1964a) indicate that the general viscoelastic model will satisfactorily reproduce uncompacted soil behavior for small strains. Small strains were determined to be those that would not change internal structure—cause permanent rearrangement of soil particles. Kondner and Ho (1965) matched predicted to actual behavior for a saturated clay that was strained to 10 percent. He demonstrated, as expected for a viscoelastic material, that stress-strain ratios are quite different for pulse loading situations than for constant loading conditions.

Saal and Labout (1958) illustrate the use of viscoelastic theory very well for describing asphalts. Consider the stress-strain ratio for a viscoelastic material. This ratio may be plotted as a function of stress application time in order to illustrate the behavior of a material (Fig. 1). The main point to remember from this figure is that for a viscoelastic material the stress-strain ratio decreases as the time of application of a given stress increases. In other words, a one-atmosphere pressure applied for less than one second to a viscoelastic material will result in much less strain than will the application of one-atmosphere pressure applied for several thousand seconds. Kondner (1967) presents similar information for saturated clays.

For a complete description of soil behavior, both short-time and long-time properties must be described. Vibrational or sonic radiation measurements with frequencies in the audiofrequency range (20 to 20,000 cycles per sec) can provide information primarily on short-time loading properties. The stress-strain ratio for very short-time compressional loading is Young's modulus. Even though changes in Young's modulus (or shear modulus if shear waves are used) can be followed with periodic measurements, it should be realized that it is a short loading time property that is being measured. Unless experimentally verified, relationships between long and short loading time properties are found, long loading time behavior must be measured independently. There have been attempts to develop such relationships. Strick (1967) had some success in predicting creep (long loading time stress-strain curves) in a plexiglass from wave propagation data.

Wave propagation measurements in field soils and equipment used are described by Maxwell and Fry (1968). They use a large variable frequency vibrator to produce surface shear waves and standard refraction techniques (hammer or small explosion pulses as used by Hobson and Hunter (1969)) to produce compression waves for measurement of shear and Young's modulus at field sites. The velocity of travel of the generated waves or pulses is used to calculate soil moduli.

These field techniques are plagued with a variety of problems. In agricultural soils, frequently interest is in shallow horizons only one to two ft thick. Field techniques work best on 5 to 10 ft or deeper layers that have increasing hardness (reflected in increasing travel velocity) with depth. Desir-
able precision is difficult to obtain in surface horizons. Soil water content influences wave velocity. Measurable differences in velocity between adjacent layers may be greater because of differences in water content than because of actual structural differences between the layers. Soils with low bulk densities have low wave velocities and high energy loss or wave attenuation. Such soils are easily deformed by large amplitude waves or pulses. Thus, the requirement for a large input pulse, because of high signal attenuation, may cause a change in the property being measured because of soil compaction by the pulse.

The same principle that is used in field measurements—timing of a pulse or wave velocity—can be used in a laboratory on prepared or undisturbed samples where water content can be controlled. The problem of sample deformation during measurement still remains.

A resonant frequency technique for measuring Young's modulus is also available for laboratory use. Stevens (1966) and Rickman (1970a) use the resonant column technique on soil columns and describe the equipment used for measurements. The technique utilizes low amplitude mechanical vibrations which permit nondestructive testing of sample columns. Fig. 2 provides a schematic of the test equipment used for such measurements. A cylindrical soil sample is vibrated at low amplitude from the bottom at different frequencies until the resonant frequency of the column is found. The resonant frequency is used to calculate Young's modulus for the column. Lee (1963) has provided a theoretical analysis of this resonant column situation with viscoelastic theory. The elastic parameter, Young's modulus, or the viscoelastic parameter, called a complex Young's modulus, can be measured with the same equipment. The complex modulus provides a description of the material response to steady-state vibrational loading. The complex modulus value is dependent upon the vibrational frequency. At low amplitude (1/10 g or less acceleration at the sample base) and "high" (100-1000 Hz) frequency, the values approach Young's modulus. At high amplitude, energy loss and sample compression require consideration of other factors (Kondner 1967).

There are techniques for determining complex modulus at low frequencies so that a portion of the loading-time range can be described by a complex modulus (Stevens 1966). Also, Kondner and Ho (1965) present a procedure to compute complex moduli from creep test data. At lower frequencies the complex portion of the modulus becomes larger, reflecting the increased percentage of energy lost within the soil with longer loading times.

DISCUSSION

Only Young's modulus rather than the complex modulus is reported here for soils because at the low vibrational amplitudes, the energy loss portion of the measurements caused complex moduli to be less than 5 percent different than Young's moduli.

Presented in Fig. 3 are values for Young's modulus of unconfined 7.5-cm diameter, 15-20-cm high columns of three soils. They were measured on duplicate samples over several wetting and drying cycles. In order to be sure the reader is oriented with the property displayed, examine Fig. 1. Young's modulus for the soils varies up and down the vertical axis of Fig. 1 as a function of water content. The Young's modulus values in Fig. 3 do not at this time reflect any of the long-time loading properties of the soils.

The soils used for the data reported here were a Blackhawk silt loam, 0- to 3-in. surface sample of a profile similar to profile 58 of the Seventh Approximation (Staff, 1960); a Portneuf silt loam, a highly productive soil from south-central Idaho; and a Chesterfield loamy sand from Alabama. The preparation and measurement procedure to collect this data is described by Rickman (1970a). The curves in Fig. 3 provide a quantitative illustration of some mechanical properties of the three soils. The grain size of the sand fraction of the Chesterfield loamy sand is primarily a medium sand; the non-sand fraction is nearly all clay. The soil has an appreciable modulus value even when near saturation. As the soil dries, it hardens rapidly. Curve B also exhibits the rapid hardening with decreased water content, but for a different reason. Blackhawk silt loam has a very low organic matter content and some sodium on the exchange lattice. It loses all of its rigidity as it is wet to near saturation. This soil would have severe crusting problems under any agricultural situation. Curve P is from a Portneuf silt loam which has a higher organic matter content than the Blackhawk silt loam and is calcium saturated. The Portneuf silt loam retains a finite modulus value near saturation and does not harden as rapidly as the other two soils when it dries.

On the basis of some assumptions about the manner in which root growth may be modelled (Rickman, 1970b), a Young's modulus value of approximately 100 dekabars can be used to roughly indicate the conditions under which root elongation stops. Both the Chesterfield loamy sand and the Blackhawk silt loam have measured moduli greater than 100 dekabars while the Portneuf silt loam does not. The effects of soil strength on root growth might be estimated from Young's modulus data, but accurate predictions should not be expected because Young's modulus is a short-time loading measurement. Long-time loading measurements of soil properties would be required to obtain maximum accuracy in predicting root growth.

Families of stress-strain curves that provide a complete mechanical description of a soil can be projected from more extensive data on the long-time loading response of sands and clays by Kondner (1967) and Stoll (1967). Such curves are admittedly speculative but seem logical on the basis of data present in the literature. Such curves provide a fairly complete picture for expected stress-strain versus time curves for a soil

FIG. 2 Equipment and sample arrangement for measurement of soil column resonant frequency for calculation of Young's modulus. (VTVM is a vacuum tube volt meter).

FIG. 3 Young's modulus versus water content for confined columns of three soils.
like the Chesterfield loamy sand of Fig. 3. When dry, soils with such uniform large particle sizes, behave elastically. Only after the soil is quite wet does the stress-strain ratio decrease for large loading times. Chae (1967) explains some conflict of reports on the effect of water on compressional stress-strain response of sands. Confined, clean, coarse sand have changes in their moduli as small as 3 to 9 percent between dryness and saturation. Mixed grain sizes, the fine sands, and unconfined sands exhibit larger changes of moduli with water content. Contrast the behavior illustrated in Fig. 4 with expected behavior of a soil like the Blackhawk silt loam as shown in Fig. 5. Such a soil behaves elastically when dry but with increasing water content the time-dependent nature of the stress-strain is more and more important. Near saturation this soil has essentially fluid characteristics. Kondner (1967) provides a detailed analysis of such a soil based on stress-strain behavior of wet clay.

Only part of the information needed to accurately construct curves such as those in Fig. 4 and 5 may presently be obtained from sonic testing techniques. There are attempts, as mentioned earlier, to relate wave propagation measurements to long-time loading responses in

FIG. 4 Possible set of stress-strain versus loading time curves for a loamy sand such as the Chesterfield loamy sand.

Young's moduli measured with sonic techniques for uncompacted soils provide part of the information needed to describe the mechanical behavior of an agricultural soil over a wide range of water contents. Soil modulus values vary with water content in a manner that permits qualitative prediction of soil behavioral properties from visual inspection of Young's modulus versus water content curves. If stress-strain data for long-time loads are available, they can be combined with modulus data to form a family of stress-strain ratio versus time curves to characterize a soil mechanical response over a wide range of water content. Stress-strain ratio versus time curves can clearly show the different types of mechanical response of various soils. Sonic measurements presently have to be supplemented with long-time static tests to obtain a complete picture of soil mechanical behavior.

**SUMMARY**

Young's moduli measured with sonic techniques for uncompacted soils provide part of the information needed to describe the mechanical behavior of an agricultural soil over a wide range of water contents. Soil modulus values vary with water content in a manner that permits qualitative prediction of soil behavioral properties from visual inspection of Young's modulus versus water content curves. If stress-strain data for long-time loads are available, they can be combined with modulus data to form a family of stress-strain ratio versus time curves to characterize a soil mechanical response over a wide range of water content. Stress-strain ratio versus time curves can clearly show the different types of mechanical response of various soils. Sonic measurements presently have to be supplemented with long-time static tests to obtain a complete picture of soil mechanical behavior.

**References**