Hydraulic and Geometrical Relationships of LAY-FLAT IRRIGATION TUBING

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HEADING LOSS IN FLOW CONDUITS

The shape of the conduit has a significant impact on the frictional loss. The smoothness of the interior surface of the conduit affects the flow resistance. The smoother the surface, the lower the frictional loss.

**Problem**

**File**

1. **Definition of Head Loss**
2. **Solution**
3. **Discussion**

**Diagram**

[Diagram of a conduit with labeled areas and dimensions]
PARALLEL BOUNDARIES

The flow regime for uniform flow through lay-flat tubing ranges from that for rigid pipe to that for flow through parallel-boundary conduits. Rigid pipe and parallel boundaries represent the extreme conduit shapes. Therefore, expressions for the friction coefficients for flow through parallel boundaries, corresponding to those for pipe flow, are presented. The Karman-Prandtl equation for the resistance factor for smooth parallel boundaries of infinite extent is

\[ \frac{1}{\sqrt{f}} = 2.03 \log_{10} R \sqrt{f} - 0.47. \]  

(7)

The equation for the parallel rough-boundary coefficient is

\[ \frac{1}{\sqrt{f}} = 2.03 \log_{10} \frac{B/2}{k} + 2.11. \]  

(8)

Because of insufficient data, the constants in equations 7 and 8 have not been adjusted to provide agreement with experimental measurements. The equations for pipe and parallel-boundary flow are identical except for the integration constant. The factor 2.03 was adjusted to 2 in the pipe equations, to provide better agreement with experimental measurements. Since this factor appears in the expressions for both cross sections, the effect of conduit shape upon the friction factor must be represented by the integration constant for each type of flow.

The Darcy-Weisbach equation for parallel-boundary flow is

\[ h_r = f \frac{L V^2}{2B 2g}. \]  

(9)

The diameter in equation 3 and twice the boundary spacing in equation 9 are each equal to four times the hydraulic radius for their respective cross-sectional shapes. Therefore, if the diameter and twice the boundary spacing are replaced by four times the hydraulic radius, the Darcy-Weisbach equation becomes

\[ h_r = f \frac{L V^2}{4R 2g}. \]  

(10)

for both pipe and parallel-boundary flow.

LAY-FLAT TUBING

Keulegan \(^4\) proposed equations for the friction coefficient for open channels of various cross sections in which the channel shape was represented by a shape factor term in the expression. To the authors' knowledge, however, no equations have been developed for closed conduit flow covering the entire shape range between round and rectangular.

Since the general head-loss equation, equation 10, is the same for both pipe and parallel-boundary flow and since the expressions for \( f \) are the same at each end of the flow regime except for the constants embodying the shape effect, the friction loss through tubing may also be represented by this equation, inasmuch as lay-flat tubing represents a conduit that varies in shape from round to rectangular. It is necessary, however, to modify the friction coefficient to include the effects of shape change. When equation 10 is used for lay-flat tubing, \( f \) may be considered as consisting of two parts: (1) \( f' \), the friction coefficient for an equivalent flow through a pipe of the same diameter, and (2) a shape factor \( \beta \), which corrects \( f' \) for the effects of tube shape. In other words

\[ f = \beta f'. \]  

(11)

Thus, the frictional head loss for uniform flow through lay-flat tubing may be represented by

\[ h_r = \beta f' \frac{L V^2}{4R 2g} \]  

(12)

in which \( \beta \) and \( R \) are the shape factor and hydraulic radius, respectively, for a given cross-sectional shape. The friction coefficient \( f' \) may be obtained from a conventional pipe resistance diagram of \( f \) versus Reynolds number. The shape factor \( \beta \) represents the ratio \( f/f' \) and is evaluated empirically from flow test data obtained in this study.

HYDROSTATIC TESTS

As pointed out by Hansen \(^7\), a study of the hydraulic properties of lay-flat tubing must be preceded by a study of the tube cross-sectional shape and area. Since the head loss and flow capacity of a conduit are a function of the conduit geometry, it is necessary, in studying tubing hydraulics, that its geometry at all pressure heads be known. It is also necessary to know the pressure head and velocity at a given location.

It would be difficult and generally impractical, because of the nature of the material, to install instruments to measure the flow velocity. If the cross-sectional area were known, the mean flow velocity could easily be determined from the continuity equation \( Q = a V \) at any point along the tube. As further indicated by Hansen, it may not be practical in some cases to attach pressure gages or taps to determine the piezometric pressure head at a location. The tubing geometry might be utilized for these purposes if data were available that related the tube geometry to the pressure head and cross-sectional area. These relationships were determined from hydrostatic tests made in the laboratory as herein described. The data are plotted in


dimensionless ratios and are, therefore, applicable to tubes of different diameters.

**EXPERIMENTAL TESTS**

**Tubing Material Tested**

Hydrostatic tests were made on tubes of different materials and sizes (table 1) in which the tube cross section was traced at different hydrostatic pressures. Two types of tubing were used: (1) a thin-walled vinyl chloride plastic and (2) a comparatively thick-walled butyl rubber tube. A 3-inch-diameter vinyl chloride tube was supported by rayon, whereas the rest of the vinyl chloride plastic tubes were supported by nylon. The butyl rubber tubes were also supported with nylon. The vinyl chloride tubes having a wall thickness of 0.01 inch were very flexible. As a result, the resistance to bending offered by the walls of these tubes would be very slight and, therefore, have little effect upon the area relationships. The butyl rubber tubes, however, with a wall thickness of 0.065 inch would offer a greater resistance to bending and were tested to determine the influence of tubing wall thickness upon the shape characteristics.

**Table 1.—Description of lay-flat tubing used in hydrostatic tests**

<table>
<thead>
<tr>
<th>Tubing material</th>
<th>Nominal diameter</th>
<th>Actual diameter</th>
<th>Tubing wall thickness</th>
<th>Supporting structure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inches</td>
<td>Inches</td>
<td>Inch</td>
<td></td>
</tr>
<tr>
<td>Polyvinyl chloride plastic</td>
<td>3</td>
<td>2.90</td>
<td>0.017</td>
<td>Rayon (23 × 23 thread per inch)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.17</td>
<td>0.010</td>
<td>Nylon</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5.00</td>
<td>0.010</td>
<td>Do.</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5.00</td>
<td>0.014</td>
<td>Nylon (52 × 30 leno weave)</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>11.70</td>
<td>0.010</td>
<td>Do.</td>
</tr>
<tr>
<td>Butyl rubber</td>
<td>4</td>
<td>3.92</td>
<td>0.065</td>
<td>Do.</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5.95</td>
<td>0.065</td>
<td>Do.</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>8.40</td>
<td>0.065</td>
<td>Do.</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>11.60</td>
<td>0.065</td>
<td>Do.</td>
</tr>
</tbody>
</table>

**Testing Procedure**

The tube in which the cross section was to be measured was supported on a sheet of plywood leveled in each direction. Fittings were attached to the tube in such a manner that water could be admitted to the tube and drained from it. A manometer lead was also attached to the tube and connected to a manometer from which the piezometric pressure head was measured. The tube length varied with diameter, but it was at least 10 diameters long. This allowed a minimum of 5 diameters between the end and the section where the tube cross section was traced. The effect of round end fittings upon the cross-sectional shape was thus eliminated.

A special pantograph was constructed and used to trace the cross-sectional shape of the tubes at different hydrostatic pressures. With this device, the cross-sectional shape of the tube was traced on a sheet of paper as a pointer was moved around the outside perimeter of the tube. The pantograph and the experimental installation are shown in figure 2.

![Figure 2.—Pantograph and experimental setup for tracing the cross section of lay-flat tubing.](image)

Tracings were made at various hydrostatic pressure heads so that a plot of pressure versus cross-sectional area resulted in a well-defined...
curve over the entire shape range from lay-flat to round. Duplicate tracings were made of the tube cross-sectional area at each pressure head.

The vertical-height dimension of the tube was measured for each tracing by a point gage mounted on the pantograph support stand directly over the center of the tube. The horizontal-width dimension of the larger tubes was determined by measuring the distance between two T-squares set adjacent to the tube on each side. The width of the smaller tubes was measured with calipers. Since the tube perimeter changed with the pressure, it was also measured.

Similar measurements were also made to determine the height, width, and area relations of tubing filled with fluids of different specific weights. Tracings were made on an 8-inch vinyl tube with each of the following fluids:

1. Brine, specific weight = 72.5 lb./cu. ft.
2. Tap water, specific weight = 62.5 lb./cu. ft.
3. Kerosene, specific weight = 50.3 lb./cu. ft.

**Data Analysis and Presentation**

The areas of the cross-sectional tracings were determined with a planimeter. A pressure head vs. area curve was drawn from the average area of the duplicate tracings for each tube. The data, in general, defined a smooth curve, and values for determining the tube cross-sectional area at various hydrostatic pressure heads were taken from these curves. A typical curve for a given tube is shown in figure 3. Since the perimeter changed because of elongation of the tubing material when under pressure, it was necessary to measure the perimeter of the tube section for each tracing. The measured perimeter was then plotted versus the static head for each tube, as shown in figure 3. The perimetric value used to calculate the full-round tube area, A, was taken from this curve. The ratio, a/A, of the tube area at less than full round to that at full round was computed from the area a, taken from the head-area curves, and the calculated value of A.

The individual pressure head-area curves, such as the one shown in figure 3, were generalized for tubes of all diameters when the data were plotted in dimensionless ratios. This was accomplished by plotting the pressure head as H/D and the area as the ratio a/A. In a similar manner, data representing all of the tube geometrical relations were plotted in dimensionless ratios and, therefore, are applicable to tubes of different sizes. The tube height, width, area, height-width ratio, and hydrostatic head were represented by the ratios d/D, w/D, a/A, d/w, and H/D, respectively, where D is the diameter of a tube when full round. Symbols representing tubing geometry are shown in figure 1.

**Vinyl Chloride Plastic Tubes**

The parameters H/D, d/w, and d/D for the vinyl chloride plastic tubes are simultaneously related to the tube area in figure 4. The area-tube width relation for this tubing is represented by the curve shown in figure 5. When related to tube width, the area of the
a log-log plot of $a/A$ vs. $d/D$ directly did not yield a straight line, because the vertex of the parabola was not at the origin. It was necessary, therefore, to adjust the data by subtracting the area-tube height ratios from 1 to obtain a straight line relationship on the log-log plot. The general form of the equation for the line is $y=ax^n$ in which $a=1$ and $b=1.84$. The relationship, therefore, is

$$1-a/A = (1-d/D)^{1.84}$$  \hspace{1cm} (13)

or

$$a/A = 1 - (1-d/D)^{1.84}$$ \hspace{1cm} (14)

The term $(1-d/D)^{1.84}$ was expanded by the binomial theorem

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots$$ \hspace{1cm} (15)

in which

$$(a+b)^n = (1-d/D)^{1.84}$$ \hspace{1cm} (16)

The term expanded became

$$(1-d/D)^{1.84} = 1 - 1.84(d/D) + 0.773(d/D)^2 + 0.0412(d/D)^3 + 0.0052(d/D)^4 + \ldots$$ \hspace{1cm} (17)

This expression substituted in equation 14 yielded

$$a/A = 1.84(d/D) - 0.773(d/D)^2 - 0.0412(d/D)^3 - 0.0052(d/D)^4 - \ldots$$ \hspace{1cm} (18)

An empirical expression was also determined for the area-tube width relationship in which the parameters $(1-a/A)$ and $(w/D-1)$ on a log-log plot yielded approximately a straight line (fig. 7). The straight line does not fit the data at the end of the curve where the end point value for the extreme lay-flat shape limit is $\pi/2$. However, this is not of practical significance, since this end of the curve represents the tube when empty. The equation of this line is

$$(1-a/A) = 2.62(w/D-1)^{1.96}.$$  \hspace{1cm} (19)

The term $(w/D-1)^{1.96}$ was expanded by the binomial theorem in which

$$(a+b)^n = (w/D-1)^{1.96}. \hspace{1cm} (20)$$

This expression when expanded and substituted into equation 19 yielded

$$a/A = 1 - 2.62(w/D) + 5.06(w/D)^2 - 2.35(w/D)^3 + 0.97$$

$$-0.055(w/D)^{-1.97} - 0.015(w/D)^{1.97} - 0.0061(w/D)^{-2.97} - \ldots$$ \hspace{1cm} (21)

The scatter of points at the lower end of the curves shown in figures 6 and 7 is exaggerated by the expanded scale of the logarithmic

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Relation of tube height to the area on logarithmic plot for lay-flat polyvinyl chloride plastic tubing.}
\end{figure}

3-inch tube did not follow the same curve as the other tubes, because of its greater wall rigidity. The material from which this particular tube was fabricated was less flexible than that for the other vinyl tubes. Because of the flexibility and slight resistance to bending offered by the walls of the vinyl chloride plastic tubing 4 inches and larger in diameter, the data for these tubes represent most accurately the true pressure head and shape-area relationships.

Although these relations are given for water, they are also applicable for other fluids, provided the pressure head is expressed in terms of the particular fluid used. The effect of fluids other than water on tubing geometry is discussed later (p. 18).

An empirical mathematical expression was determined for the area-tube height relationship. This was plotted as $(1-a/A)$ vs. $(1-d/D)$, which on logarithmic coordinate paper yielded a straight line (fig. 6). Although the area-tube height relationship is parabolic,
and-smaller butyl tubes was less than that of the vinyl tubes for a given hydrostatic head or tube height.

This area reduction was caused by a fold or crease that formed at the two edges of the butyl rubber tubing when it was packaged and stored in a lay-flat position. Since the material was relatively stiff, the creases were semipermanent and remained until the fluid pressure inside the tube was sufficient to straighten them out. The area, particularly at low heads, was therefore reduced. When the tube approached a full-round shape, the head was sufficient to cause the creases to disappear and the cross-sectional area then approached that of a polyvinyl chloride plastic tube. The difference was more pronounced in tubes of small diameter than with those of a large diameter and also with new tubes as compared with those that had been in use for a time. With continued use, the creases would become less pronounced or disappear. The area ratio of the butyl rubber tubes would then approach the same value as that for the thin-walled tubes when related to the tube height and the hydrostatic head.

The results showed that the cross-sectional area when related to tube width varied with the type of tubing and also with the diameter of the butyl tubing. Figure 8 shows data from the thick-walled butyl tubing superimposed upon the area-tube width curve for the vinyl tubing that was shown in figure 5. The area ratio for the very flexible thin-walled vinyl chloride tubes having the same wall thickness was represented by a single curve. A separate curve was required to represent the area ratio of each of the butyl rubber tubes tested. These differences in the area, when related to the tube width, were apparently caused by the difference in tubing wall stiffness and also by the edge creases. As these effects decrease, it would be expected that all tubing of like material could be represented by the same curve and that the curves would approach more nearly that of a completely flexible tube.

For the tubes tested, the test results indicate that the data, when expressed in dimensionless ratios, may be represented by the same curves, whether or not the data are expressed in ratios representing inside or outside dimensions. However, for 4 to represent the inside area of the tube cross section, the parameters d/D, w/D, and A, used in its determination, must also represent inside dimensions. This is particularly true of such thick-walled tubes as the butyl rubber tubes tested.

Alternative Method to Determine Tube Area

The cross-sectional area of a tube may be determined from the product of the tubing's height-width dimensions multiplied by some factor. As the tube becomes circular, the height and width dimensions each approach the diameter and the factor approaches π/4. With little or no hydrostatic head, the tube cross section approaches that of a rectangle for which the factor approaches a value of unity. Quantitatively, this factor is a dimensionless number representing the ratio a/ωd and is shown plotted versus tube shape, as represented by the height-width ratio (fig. 9). This curve was determined from the polyvinyl chloride plastic tube tests and will not apply to butyl
Figure 3—The ratio of depth to the height-vs-width ratio for lay-flat polyvinyl chloride plastic tubing.

POLYVINYL CHLORIDE

Figure 6—Area-tube width relationship of butyl rubber tubing superimposed on the corresponding curve for nylon-supported poly butyl rubber tubing.
rubber tubing unless the side creases in the tubes have disappeared either through use or because of the fluid pressure in the tube.

Effect of Fluid Density Upon Tube Shape

Corresponding data from tests conducted with fluids of different specific weights were also represented by the curves shown in figures 4, 5, and 9 when the hydrostatic head was expressed as a column height of the respective fluid. When the pressure head of the different fluids was expressed as the height of a column of water, however, there was a difference in the head-area relation, as shown in figure 10. In contrast to head-area curves, the individual curves representing the oil and brine fluids, in effect, are pressure-area curves for a particular fluid but generalized as to tube size. The three curves of figure 10 indicate that the greater the fluid density, the greater the pressure required to yield a given cross-sectional area.

The pressure head parameter \( H/D \) is an abbreviated form of the parameter \( \frac{P}{\gamma D} \). When the tube area, represented by \( a/A \), is plotted against \( \frac{P}{\gamma D} \), the resulting curve is a generalized pressure-area curve for fluids of different density. When the parameter \( H/D \) is used in place of \( \frac{P}{\gamma D} \), it is implied that \( H \) represents the hydrostatic head of the respective fluid. When it does, the \( H/D \) vs. \( a/A \) curve (solid line curve of fig. 10) is a generalized curve for different fluids. This is not true of the broken line curves of figure 10, since \( H \) for them does not represent their respective fluid but is expressed as a column height of water.

The variation in density of waters used for irrigation is relatively small. Therefore, when lay-flat tubing is used for irrigation, the effect of fluid density upon tube shape is so small that it becomes a matter of interest only and is not of practical significance.

**DISCUSSION OF RESULTS**

From the data that have been presented, it is possible to determine the cross-sectional area and describe the tube shape, as represented by the height-width ratio, for lay-flat tubing of any degree of roundness. To do this, the full-round diameter must be known and any one of the following: tube height, tube width, or both, or the hydrostatic head. With the area ratio and the full-round area known, the tube cross-sectional area may be readily obtained.

With the area known, the mean flow velocity and the hydraulic radius are easily determined. The hydraulic radius may be obtained directly from the area ratio, since

\[
\frac{a}{A} = \frac{r}{R}
\]

where \( r/R \) is the ratio of the hydraulic radius for a tube at any shape to the hydraulic radius of a full-round pipe. Tube width, because of the influence of tubing wall thickness and stiffness, is not the best...
single parameter to use in determining the tube area. The $H/D$ vs. $a/A$ curve (fig. 10) becomes very steep when the tubing approaches a circular shape. Therefore, special care must be exercised when the tube area is determined from this region of the curve.

The tube height is a good parameter from which to determine the area ratio throughout the entire shape range. For most applications, the curve in figure 4 showing the relation of hydrostatic head, tube height, area, and height-width ratio will find the most frequent use. Since these data were obtained with the tubing supported by a flat surface level in a direction perpendicular to the tube length, they are most applicable to tubes supported in this manner. This represents an ideal condition; further research will be needed to determine the deviation therefrom when a tube is supported by a rough or side sloping surface.

The cross-sectional area of tubing, 4 to 16 inches inside diameter, was computed for different degrees of tubing roundness and at different head-diameter and height-diameter ratios (table 2). These values are applicable to all types of lay-flat tubing when supported on a relatively flat surface except thick-walled tubing, such as butyl rubber approximately 6 inches and smaller in diameter. The cross-sectional area of this tubing may be slightly less than that indicated in the table.

FLOW TESTS

The flow-testing phase of this study was undertaken to determine the effect of tubing shape upon the friction coefficient and to obtain data from which the shape factor in equation 12 could be evaluated.

EXPERIMENTAL PROCEDURE

Flow tests were conducted with 6- and 8-inch nylon-supported polyvinyl chloride plastic experimental tubing. An 80-foot, slope adjustable ramp was used to support the tubes on which the flow tests were made (fig. 11). The tubing at the intake end was fastened to a 12-inch water supply line. Water from the tube discharged freely into an open flume at the lower end of the ramp. Flow rates of 1 c.f.s. and greater were measured with a 12-inch flowmeter installed in the supply line, whereas lesser flow rates were determined with a small weighing tank.

Coreless valve stems were secured to the bottom of the tube at 5-foot intervals along the test section. Clear plastic tubing connected the valve stems to a manometer board from which the piezometric pressure head at each 5-foot station was measured (fig. 12). Geometrical dimensions of the tube were measured during the tests. The tube height was measured with a point gage mounted on a stand that straddled the tube (fig. 13). The stand was also fitted with a scale with which to measure the horizontal width of the tubing. The perimeter varied slightly along the tube and, therefore, was measured with a small metallic rule at each station.

Tests were made on each tube, with the tube resting on a 0-, 1/2-, 1-, and 2-percent slope. Runs were also made at zero slope with a 6-inch tube full round.
Nonuniform Flow

For nonuniform flow, equation 10 was modified to include the accelerative force that accompanies nonuniform flow. The forces that act upon a volume of water flowing nonuniformly are shown in figure 14, in which the accelerative force is represented by the equal and opposite inertial force. The forces maintaining flow are the pressure force, \( p_a a_1 - p_a a_2 \), and the component of weight acting in the direction of flow \( \gamma S_a a \) minus the accelerative force \( \rho V a \Delta V \). The opposing force is the shear force \( \tau_s PL \). For equilibrium, these forces must equal zero. Summation in the \( x \) direction gives

\[
p_a a_1 + \gamma S_a a - p_a a - \tau_s PL - \rho V a \Delta V = 0.
\]

(23)

The boundary shear stress \( \tau_s \) may be expressed in terms of the quantity \( \rho V^2/2 \) and the Darcy-Weisbach resistance coefficient \( f \); thus

\[
\tau_s = \frac{1}{4} \frac{\rho V^2}{2} = \frac{f}{8} \rho V^2.
\]

(24)

Therefore, if \( \tau_s \) is replaced by \( \frac{f}{8} \rho V^2 \) and \( p \) by \( \gamma h \), equation 23 becomes

\[
\gamma h a_1 - \gamma h a_2 + \gamma S_a a = \frac{f}{8} \rho V^2 PL + \rho V a \Delta V
\]

(25)

from which

\[
f = \frac{8 \gamma (h a_1 - h a_2 + S_a a)}{\rho V^2 PL} = \frac{8 \Delta V R}{VL}
\]

(26)

in which \( R \) was substituted for \( a/P \). A sketch of \( h \) and the pressure forces that act on the ends of a section of fluid flow is shown in figure 15. The expression for head loss for nonuniform flow from equation 26 is

\[
h_f = \frac{f}{4R} \frac{L V^2}{2g} + \frac{V a \Delta V}{g}
\]

(27)

This equation is valid for short length increments, but if it is used over a long length-increment, an error would be introduced. When the head loss is expressed in this form, it is assumed that

\[
p_a a_1 - p_a a_2 = \Delta p a
\]

where \( p \) is the average pressure intensity acting on the average area \( a \) in which

\[
a = \frac{a_1 + a_2}{2}.
\]

See figures 14 and 15. This assumption is not exactly correct where there is a large difference between the area \( a_1 \) and \( a_2 \). The friction coefficient was determined with equation 10 in which

\[
f = \frac{8Rgh}{LV^2}.
\]

(22)
The shape factor $\beta$ is related to the tube shape, represented by the height-width ratio, and is shown by the curve in figure 16. The shape factor represents the effect upon head loss of the change in both velocity distribution and relative roughness caused by shape change.

The broken line curves shown in figure 16 indicate the trend of the curve beyond the data. Since a zero value of the ratio $d/w$ represents a parallel boundary conduit of infinite extent, each end point curve was determined by evaluating the shape factor for parallel boundaries. This was accomplished by determining the ratio of the $f$ value for parallel boundary flow to that of pipe flow at a given Reynolds number for different boundary spacings.

A definite end point at $d/w = \infty$ for the curve cannot be found, since the $f/f'$ ratio varies with Reynolds number. This variation results from the fact that the family of curves on the resistance diagrams are not parallel to each other from one value of Reynolds number to another. The Colebrook-White transition equation (equation 6) was modified for use with parallel boundary flow. This function, used to define $f$ in the transition region between smooth and rough parallel boundaries, was:

$$\frac{1}{f} = 2.11 - 2 \log_{10} \left[ \frac{k}{c_D R \sqrt{f}} \right]$$

A resistance diagram for parallel boundaries was constructed by plotting values of $f$, computed with equations 7, 8, and 29, versus Reynolds number for different values of relative roughness. The $f$ values for parallel boundary flow used in the ratio $f/f'$ were taken from this resistance diagram.

Also shown in figure 16 is the calculated theoretical shape factor for laminar flow through an elliptical-shaped conduit as given by Lamb\textsuperscript{*} in which

$$\beta = \frac{2dv}{d^2 + w^2}.$$  

This equation is included merely to show a comparison between the shape factor for laminar and turbulent flow through similarly shaped conduits.

With values of the shape factor $\beta$ known, equation 12 may now be used to represent the frictional head loss for uniform flow through lay-flat tubing.

**DISCUSSION**

**Design Equations for Friction Loss**

The head loss for uniform flow through a tube may also be determined in terms of the loss for the same flow through a round rigid pipe of the same size and relative roughness. Since

$$V^2 \frac{4R}{4} = \int_0^\infty \left[ \left( \frac{A}{a} \right) V_p \right] \left( \frac{a}{A} \right) \frac{V_p^2}{D}.$$  

equation 10 may be expressed in the following form:

\[ h_r = \left( \frac{A}{d} \right) f' \frac{L V_p^2}{D^2} \]  

(31)

A convenient form, therefore, in which to express equation 12 for the head loss through lay-flat tubing is

\[ h_r = \beta \left( \frac{A}{d} \right)^{n+1} \frac{K L V_p^2}{D^m} \]  

(32)

The factor \( \beta \left( \frac{A}{d} \right)^{n+1} \) is constant for any given tube shape and is related to the height-width ratio and the head-diameter ratio by the curve shown in figure 17. The loss for uniform flow through a tube may be determined by multiplying the loss for the same flow through a round pipe by the factor \( \beta \left( \frac{A}{d} \right)^{n+1} \). Equation 32 will probably be the simplest and most practical form to use in designing and evaluating lay-flat tubing for field application.

If the head loss were to be expressed in the form of equation 1, the equation applicable to lay-flat tubing would be

\[ h_r = \beta \left( \frac{A}{d} \right)^{n+1} \frac{K L V_p^2}{D^m} \]  

(33)

in which values of \( K \), \( V_p \), and \( D \) are those applicable to the same flow through a pipe of the same size and roughness as the tube when round.

Use of the friction coefficient \( f' \) may in some cases introduce a small error, since it is determined from the Colebrook-White transition curve on the pipe resistance diagram. This curve is for flow through a pipe of commercial or nonuniform roughness. For tubing that is not hydraulically smooth, the absolute roughness of the material from which it is fabricated will, in most cases, be more uniform than that of commercial pipe. Because of this, the \( f' \)-Reynolds number relationship may depart slightly from the transition curve toward the curve pattern obtained by Nikuradse\(^{19}\) in his experiments on pipe of uniform roughness. In most cases the error will likely be very small.

**Relative Roughness**

As noted previously (p. 2), the tubing relative roughness is affected by a shape change and is difficult to define. For a tube shape in between the two limits of pipe and parallel boundaries, the depth of flow midway between boundaries is different in the width direction from that in the height direction. The problem, therefore, is to represent the two flow depths by a single dimension that will be valid from one shape limit to the other. Four times the hydraulic radius

---

\(^{19}\) See footnote 5.
is often used to represent this characteristic length. This is valid for open channel flow, but for closed conduits, use of the hydraulic radius introduces an error of two as the cross section approaches the parallel-boundary shape limit. At this limit, the actual relative roughness is two times as great as that indicated when four times the hydraulic radius is used in its determination.

The relative roughness for a circular shape is represented by

$$\frac{\varepsilon}{D} = \frac{\varepsilon}{4R},$$

whereas for parallel boundaries it is represented by

$$\frac{\varepsilon}{B} = \frac{\varepsilon}{2R}.$$

In the absence of more precise information, the tube height $d$ would likely be the best parameter from which to compute the actual tubing relative roughness. Use of this dimension would give the correct value when the tube is full round and also as it approaches a rectangular shape, even though it may not be exactly correct at shapes in-between these two limits.

The effect of relative roughness, as related to tube shape, upon the friction coefficient is not separately evaluated in this bulletin but is included in the shape factor $\beta$. The hydraulic design is thereby made easier because the roughness value used in determining $f'$ is constant for each tube irrespective of tube shape. If the effect of relative roughness upon head loss were to be represented by the friction coefficient $f'$ instead of $\beta$, $f'$ would be determined from a resistance diagram in which the tubing relative roughness would be computed from a representative tube dimension such as tube height.

In this case, a different shape factor would be required, since the relative roughness effect would be included in $f'$ rather than in the shape factor.

**Further Study Needed**

The effect of ground surface roughness upon flow through lay-flat tubing may exceed the shape effects. This problem has not yet been studied. The initial laboratory investigation reported herein needs to be supplemented by further study in the field.

The nonuniform flow phase of the problem and the effects of accelerated flow near the end of the tube need further study.

Additional information is needed to determine more fully the requirements and performance characteristics of lay-flat tubing materials. Further development of tubing outlets and couplings with their hydraulic characteristics is needed.

**Discharge Diagrams**

Typical diagrams relating the discharge to the head loss for irrigation tubing of different diameters and degrees of roundness are shown in figures 18 to 22. These diagrams show the influence of tube shape...
upon the discharge for tubes having a boundary varying from smooth to 0.003 inch equivalent roughness. The curves in these figures were computed with equation 32. To use the diagram for tubes at less than a full-round cross section, it is necessary to know the pressure head in the tube. In designing for uniform flow on sloping ground, this may be assumed, in most cases, to be the same as the head on the tube at the intake end. The capacity of a tube flowing at less than full-round may be increased by providing for additional head to bring the tube to a more round cross section; a margin of safety may therefore be provided.
Care is needed in using the discharge diagrams for tubing on rough ground surfaces. The friction coefficient does not include the effect of ground surface roughness upon the head loss. Therefore, a margin of safety must be provided by the designer until adequate information is available to evaluate accurately this effect.

Examples are presented to illustrate how figures 18 to 22 may be used in the design of tubing on a smooth ground surface. Minor head losses such as entrance and coupler loss have been omitted to simplify the illustrations.
From figure 22 the head loss would be 4.8 feet per 1,000 feet or a slope of 0.0048.

Problem: If we assume flow is uniform, what size tube will be required to carry 3 c.f.s. at a ground slope of 0.5 percent with a 2-foot head available at the intake. Try a 12-inch tube:

\[
\frac{H}{D} = \frac{24}{12} = 2.0
\]

Head loss per 1,000 feet equals 5 feet. From figure 22 the discharge equals 3.25 c.f.s. A 12-inch tube therefore will be adequate (if tubing boundary roughness is assumed equal to 0.003 inch).

SUMMARY

STATIC TESTS

The hydraulic properties of lay-flat irrigation tubing are closely related to the shape and cross-sectional area of the tube. These characteristics are in turn dependent upon the fluid pressure head inside the tube. A study was undertaken to obtain data from which the pressure-shape-area relations could be determined. The cross section of tubing at various hydrostatic pressure heads was traced with a special pantograph. Tracings were made on supported polyvinyl chloride plastic and butyl rubber tubes of different diameters. The area of the cross-sectional tracings was measured and related to the hydrostatic head, tube height, width, height-width ratio, and height-width product. The results, expressed in dimensionless ratios, are shown in figures 4 to 10.

The cross-sectional area of tubing 4 to 16 inches in diameter for different degrees of tubing roundness was determined and is presented in table 2.

FLOW TESTS

A study was also undertaken to determine the effect of tube shape upon the frictional head loss. Flow tests were conducted on 6- and 8-inch nylon-supported polyvinyl chloride experimental tubing. Frictional head loss for uniform flow through the tubing was expressed in the form of the Darcy-Weisbach equation in which

\[
h_f = f \frac{L V^2}{4 R 2g}
\]

The frictional head loss for nonuniform flow was expressed by a modified form of the Darcy-Weisbach equation in which

\[
h_f = f \frac{L V^2}{4 R 2g} + \frac{VAV}{g}
\]

The friction coefficient \( f \) is equal to \( f' \beta \) where \( \beta \) is a shape factor that varies with tube shape. The friction coefficient \( f' \) is for an equivalent
### Symbols and Definitions

Determine the discharge for rectangular tube of different diameters and degrees of roundness to the head loss are presented.

\[
\text{Discharge} = \left( \frac{n}{V} \right) \times \text{Product of} \times \text{Pipe}
\]

when the same flow through a round pipe of the same size and roundness, the same flow through a rectangular pipe of similar dimensions when compared to a pipe of the same size and roundness, the head loss for rectangular flow through a tube of a rectangular pipe resistance direction, determined from a conventional pipe resistance direction.

**Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Dimension</th>
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</thead>
<tbody>
<tr>
<td>R</td>
<td>Radius</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Specific weight of water = 62.4 lb/ft²</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Pipe</td>
<td></td>
</tr>
<tr>
<td>H</td>
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<tr>
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<tr>
<td>D</td>
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<td>P</td>
<td>Pressure</td>
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</tr>
<tr>
<td>I</td>
<td>Intensity</td>
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